WAGES, EMPLOYMENT AND FUTURES MARKETS *

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1. Introduction

The use of financial derivatives to assess and manage exposures to various sources of risk is the norm rather than the exception in modern corporations. A *Fortune* article summarizes the characteristics of financial derivatives: "These financial innovations both warm and burn.... Love them or hate them, they're all here to stay." As financial derivatives are here to stay, corporate managers have to understand them and know how to use them to reduce risk in an effective way.

Given the real-world prominence of risk management, there have been a great many papers concerning the production and hedging decisions of a risk-averse firm under uncertainty (see, e.g., Danthine, 1978; Holthausen, 1979; Katz and Paroush, 1979; Kawai and Zilcha, 1986; Broll, Wong and Zilcha, 1999; Wong 2003). We add to this literature by extending the analysis to a standard efficiency wage model. The essential feature of the efficiency wage hypothesis is that wages enter into the production function of a firm in a labor-augmenting way. The positive dependence of work effort (or productivity) on wages can be justified on the grounds of moral hazard, turnover costs, adverse selection, and/or sociological reasons.[1] Efficiency wages differ from market clearing wages in that the latter

[1]See, e.g., Solow, 1979, 1990; Schlicht, 1978, 1992; Salop, 1979; Weiss, 1980; Akerlof, 1982; Yellen, 1984; Shapiro and Stiglitz, 1984.

serve the function of equating demand and supply in the labor market, while the former prevent workers from leaving or shirking. In this sense, efficiency wages are devised so as to control future behavior of workers and to achieve better risk-sharing between firms and workers. As advocated by Blanchard and Fischer (1986), "efficiency wage theory is surely one of the most promising directions of research at this stage."

In this paper, we present the competitive firm under output price uncertainty of Sandmo (1971), placed in a standard efficiency wage model wherein the work effort of labor depends on the wage rate set by the firm. We show that the Solow condition holds in that the equilibrium effort-wage elasticity is unity, irrespective of the availability of a commodity futures market.[2] The optimal wage rate is preference-free and independent of the underlying output price uncertainty under the efficiency wage hypothesis. It depends solely on the effort or productivity of labor such that the equilibrium effort-wage elasticity is unity. Finally, we show that the introduction of the commodity futures market affects the firm's labor decision. In particular, if the firm is sufficiently risk averse, the introduction of the commodity futures market induces the firm to hire more labor and thereby produce more output.

The rest of the paper is organized as follows. Section 2 develops a model of the competitive firm under output price uncertainty and the efficiency wage hypothesis. Section 3 examines a benchmark case wherein hedging is prohibited. Section 4 derives the firm's optimal wage and labor decisions in the presence of a commodity futures market. The final section conclude.

2. The model

Consider a variant model of the competitive firm under output price uncertainty (Sandmo, 1971). The firm produces a single commodity by using labor as the sole input. Let Q and L be the output level and the labor employment of the firm, respectively. The firm offers an endogenously determined wage rate, w, to its workers. Both the firm and workers have to pay a fraction, $t \in (0,1)$, of wages for the social security insurance. As such, the firm's

[2] The Solow condition is named by Akerlof and Yellen (1986).

total labor costs are $(1+t)wL$, and the net wage received by each worker is $(1-t)w$.

To incorporate the efficiency wage hypothesis into our model, we assume that the productivity of labor increases as wages rise. Following Malcomson (1981), we denote $e[(1-t)w]$ as the effort or productivity per worker, where $e(\cdot)$ is a strictly increasing and concave function of net wages.[3] The firm's production function is specified in a labor-augmenting way, $Q = F(\lambda)$, where $\lambda = e[(1-t)w]L$ is the labor input measured in efficiency units, and $F(\cdot)$ is a strictly increasing and concave function.

The output price uncertainty is modeled by a positive random variable, $\tilde{P}$, that denotes the spot price of the commodity at which the firm sells its output.[4] To hedge the output price risk, the firm can trade in a commodity futures market wherein each futures contract calls for delivery of one unit of the commodity at a prespecified futures price, P_0. The firm's random profit is, therefore, given by

$$\tilde{\Pi} = \tilde{P}F(\lambda) - (1+t)wL + (P_0 - \tilde{P})H, \tag{1}$$

where H is the number of the futures contracts sold (purchased if negative) by the firm.

The firm possesses a von Neumann-Morgenstern utility function, $U(\Pi)$, defined over its profit, Π, where $U(\cdot)$ is a strictly increasing and concave function, indicating the presence of risk aversion. Before the output price uncertainty is resolved, the firm chooses an employment level, L, a wage rate, w, and a futures position, H, so as to maximize the expected utility of its random profit:

$$\max_{L,w,H} \ \mathrm{E}[U(\tilde{\Pi})], \tag{2}$$

where $\mathrm{E}(\cdot)$ is the expectation operator and $\tilde{\Pi}$ is defined in equation (1). The first-order conditions for program (2) are given by[5]

$$\mathrm{E}\Big\{U'(\tilde{\Pi}^*)\{\tilde{P}F'(\lambda^*)e[(1-t)w^*] - (1+t)w^*\}\Big\} = 0, \tag{3}$$

[3]See Pisauro (1991) for the derivation of an effort supply function that possesses these properties from a moral hazard model of expected utility maximizing workers.

[4]Throughout the paper, a tilde (~) always signifies a random variable.

[5]The second-order conditions for program (2) are satisfied given the assumed properties of $U(\cdot)$, $F(\cdot)$, and $e(\cdot)$.

$$\mathrm{E}\Big\{U'(\tilde{\Pi}^*)\{\tilde{P}F'(\lambda^*)e'[(1-t)w^*](1-t)L^* - (1+t)L^*\}\Big\} = 0, \tag{4}$$

and

$$\mathrm{E}[U'(\tilde{\Pi}^*)(P_0 - \tilde{P})] = 0, \tag{5}$$

where an asterisk (*) indicates an optimal level, and $\lambda^* = e[(1-t)w^*]L^*$ is the optimal labor input in efficiency units.

3. Benchmark case without hedging

As a benchmark, we examine in this section the case that the commodity futures market is absent, which is tantamount to setting $H \equiv 0$. In this benchmark case, the first-order conditions of program (2) with $H \equiv 0$ become

$$\mathrm{E}\Big\{U'(\tilde{\Pi}^0)\{\tilde{P}F'(\lambda^0)e[(1-t)w^0] - (1+t)w^0\}\Big\} = 0, \tag{6}$$

and

$$\mathrm{E}\Big\{U'(\tilde{\Pi}^0)\{\tilde{P}F'(\lambda^0)e'[(1-t)w^0](1-t)L^0 - (1+t)L^0\}\Big\} = 0, \tag{7}$$

where a nought (^{0}) indicates an optimal level, and $\tilde{\Pi}^0 = \tilde{P}F(\lambda^0) - (1+t)w^0L^0$.

Multiplying w^0/L^0 to equation (7) and substituting the resulting equation to equation (6) yields

$$\mathrm{E}[U'(\tilde{\Pi}^0)\tilde{P}]F'(\lambda^0)\{e[(1-t)w^0] - e'[(1-t)w^0](1-t)w^0\} = 0. \tag{8}$$

Since $U'(\cdot)$, $\tilde{P}$, and $F'(\cdot)$ are all positive, equation (8) holds if, and only if,

$$\frac{e'[(1-t)w^0](1-t)w^0}{e[(1-t)w^0]} = 1. \tag{9}$$

In words, equation (9) says that the benchmark wage rate, w^0, is attained when the cost of one efficiency unit of labor, $w/e[(1-t)w]$, is minimized. This is the well-known Solow condition in the efficiency wage literature. The benchmark wage rate, w^0, is a preference-free wage rate, which is solely determined by the Solow condition that the equilibrium effort-wage elasticity is unity.[6]

Proposition 1 In the benchmark case that the commodity futures market is absent, the competitive firm under output price uncertainty optimally chooses the wage rate at which the Solow condition holds.

Proposition 1 implies that the output price uncertainty is not a determinant of the optimal wage rate under the efficiency wage hypothesis. The optimal wage rate depends solely on the effort or productivity of labor such that the equilibrium effort-wage elasticity is unity.

Rearranging terms of equation (6) yields

$$\mathrm{E}\left\{\frac{U'(\tilde{\Pi}^0)\tilde{P}}{\mathrm{E}[U'(\tilde{\Pi}^0)]}\right\}F'(\lambda^0)e[(1-t)w^0] = (1+t)w^0. \tag{10}$$

Since w^0 is determined by the Solow condition in equation (9), the benchmark labor employment, L^0, solves equation (10). Equation (10) states that L^0 is the one at which the expected marginal revenue product of labor is equal to the full marginal cost of labor, where the expected output price is evaluated using not only the underlying distribution of the random output price but also the utility function of the firm. Using the covariance operator, $\mathrm{Cov}(\cdot,\cdot)$, we can write the expected output price that is preference-adjusted in equation (10) as[7]

$$\mathrm{E}\left\{\frac{U'(\tilde{\Pi}^0)\tilde{P}}{\mathrm{E}[U'(\tilde{\Pi}^0)]}\right\} = \mathrm{E}(\tilde{P}) + \mathrm{Cov}\left\{\frac{U'(\tilde{\Pi}^0)}{\mathrm{E}[U'(\tilde{\Pi}^0)]}, \tilde{P}\right\}. \tag{11}$$

Since $\partial U'(\Pi^0))\partial P = U''(\Pi^0))F(\lambda) < 0$, we have $\mathrm{Cov}[U'(\tilde{\Pi}^0), \tilde{P}] < 0$. It then follows from equation (11) that the preference-adjusted expectation of the random output price is less than the true expectation, $\mathrm{E}(\tilde{P})$.

[6]See Schmidt-Sørensen (1990) and Lin and Lai (1994, 1997) for interesting examples where the equilibrium effort-wage elasticity may be less than unity, thereby invalidating the Solow condition.

[7]For any two random variables, $\tilde{X}$, and $\tilde{Y}$, we have $\mathrm{Cov}(\tilde{X},\tilde{Y}) = \mathrm{E}(\tilde{X}\tilde{Y}) - \mathrm{E}(\tilde{X})\mathrm{E}(\tilde{Y})$.

Proposition 2 In the benchmark case that the commodity futures market is absent, the expected output price that is preference-adjusted goes down when the competitive firm under output price uncertainty becomes more risk averse.

Proof According to Pratt (1964), the utility function, $V(\cdot)$, is more risk averse than $U(\cdot)$ if, and only if, $V(\cdot) = G[U(\cdot)]$, where $G(\cdot)$ is a strictly increasing and concave function. From Proposition 1, we know that the optimal wage rate is preference-free. Hence, the optimal labor employment, $L^\dagger$, under $V(\cdot)$ must solve

$$\mathrm{E}\Big\{G'[U(\tilde{\Pi}^\dagger)]U'(\tilde{\Pi}^\dagger)\{\tilde{P}F'(\lambda^\dagger)e[(1-t)w^0] - (1+t)w^0\}\Big\} = 0. \tag{12}$$

where $\lambda^\dagger = e[(1-t)w^0]L^\dagger$ and $\tilde{\Pi}^\dagger = \tilde{P}F(\lambda^\dagger) - (1+t)w^0L^\dagger$. Since $G''(\cdot) < 0$ and $U'(\cdot) > 0$, we have $G'[U(\Pi^0)] > (<)\ G'[U(\bar{\Pi}^0)$ for all $P < (>)\ \bar{P} = (1+t)w^0/F'(\lambda^0)e[(1-t)w^0]$, where $\Pi^0 = PF(\lambda^0) - (1+t)w^0L^0$ and $\bar{\Pi}^0 = \bar{P}F(\lambda^0) - (1+t)w^0L^0$. Hence, we have

$$\mathrm{E}\Big\{\{G'[U(\tilde{\Pi}^0)] - G'[U(\bar{\Pi}^0)]\}U'(\tilde{\Pi}^0)(\tilde{P} - \bar{P})\Big\} < 0. \tag{13}$$

Using equation (6), inequality (13) reduces to

$$\mathrm{E}\Big\{G'[U(\tilde{\Pi}^0)]U'(\tilde{\Pi}^0)\{\tilde{P}F'(\lambda^0)e[(1-t)w^0] - (1+t)w^0\}\Big\} < 0. \tag{14}$$

Comparing inequality (14) with equation (12) and using the second-order condition yields $L^\dagger < L^0$. Using equations (10) and (12), we have

$$\mathrm{E}\bigg\{\frac{V'(\tilde{\Pi}^\dagger)\tilde{P}}{\mathrm{E}[U'(\tilde{\Pi}^\dagger)]}\bigg\} = \frac{(1+t)w^0}{F'(\lambda^\dagger)e[(1-t)w^0]} < \frac{(1+t)w^0}{F'(\lambda^0)e[(1-t)w^0]} = \mathrm{E}\bigg\{\frac{U'(\tilde{\Pi}^0)\tilde{P}}{\mathrm{E}[U'(\tilde{\Pi}^0)]}\bigg\}, \tag{15}$$

where the inequality follows from $F''(\cdot) < 0$ and $L^\dagger < L^0$. □

Since $F(\cdot)$ is strictly concave and the optimal wage rate is preference-free, it follows from equations (10) and (11) and Proposition 2 that risk aversion adversely affects labor employment in both the global sense (as compared to the risk-neutral case) and the marginal sense.

4. Optimal wage and labor decisions with hedging

In this section, we resume the original case that the commodity futures market is present.

The introduction of the commodity futures market offers the firm risk-sharing opportunities. Proposition 1, however, implies that the benchmark wage rate, w^0, is unaffected by the underlying output price uncertainty under the efficiency wage hypothesis. Thus, we can immediately conclude that the optimal wage rate, w^*, in the presence of the commodity futures market must equal w^0 and obey the Solow condition.[8]

Proposition 3 Irrespective of whether the commodity futures market is present or absent, the competitive firm under output price uncertainty optimally chooses the wage rate at which the Solow condition holds.

Substituting equation (5) into equation (3) yields

$$\mathrm{E}[U'(\tilde{\Pi}^*)]\{P_0F'(\lambda^*)e[(1-t)w^*]-(1+t)w^*\}=0. \tag{16}$$

Since $U'(\cdot)>0$, equation (16) reduces to

$$P_0F'(\lambda^*)e[(1-t)w^*]=(1+t)w^*. \tag{17}$$

Equation (17) states that, at the optimum, the marginal revenue product of labor equals the full marginal cost of labor, where the random output price is locked in at the known futures price, P_0. In the presence of the commodity futures market, the optimal labor employment, L^*, is thus independent of the underlying output price uncertainty and of the firm's preferences, which is the celebrated separation theorem in the hedging literature.

Proposition 4 If the risk premium, $\mathrm{E}(\tilde{P})-P_0$, embedded in the futures price is less than the positive threshold, $-\mathrm{Cov}[U'(\tilde{\Pi}^0),\tilde{P}]/\mathrm{E}[U'(\tilde{\Pi}^0)]$, the introduction of the commodity futures market induces the competitive firm under output price uncertainty to hire more labor and thereby produce more output.

[8] This can be formally proved by multiplying w^*/L^* to equation (4) and substituting the resulting equation to equation (3).

Proof Since $w^* = w^0$, it follows from equations (10) and (17) and the strict concavity of $F(\cdot)$ that $L^* > L^0$ if

$$\mathrm{E}(\tilde{P}) - P_0 < -\mathrm{Cov}\left\{\frac{U'(\tilde{\Pi}^0)}{\mathrm{E}[U'(\tilde{\Pi}^0)]}, \tilde{P}\right\}. \tag{18}$$

Since $\mathrm{Cov}[U'(\tilde{\Pi}^0), \tilde{P}] < 0$, the threshold defined on the right-hand side of condition (18) is indeed positive. □

If the commodity futures market exhibits either contango or unbiasedness so that $P_0 \geq \mathrm{E}(\tilde{P})$, condition (18) holds immediately. Even in the case of normal backwardation, i.e., $P_0 < \mathrm{E}(\tilde{P})$, condition (18) may still hold. From Proposition 2, we know that the positive threshold, $-\mathrm{Cov}[U'(\tilde{\Pi}^0), \tilde{P}]/\mathrm{E}[U'(\tilde{\Pi}^0)]$, is higher as the firm becomes more risk averse. Thus, if the firm is sufficiently risk averse, we can conclude that the introduction of the commodity futures market always raises labor employment.

5. Conclusion

Efficiency wage models differ from standard labor market models in that wages do not necessarily clear markets. Wages are viewed as devices to encourage work effort or, more generally, to increase productivity. Firms optimally set their wages above the market clearing level, thereby generating involuntary unemployment. This paper has employed a standard efficiency wage model to address the optimal wage and labor decisions of the competitive firm under output price uncertainty (Sandmo, 1971).

Irrespective of the availability of a commodity futures market, we have shown that the Solow condition holds in that the equilibrium effort-wage elasticity is unity. The optimal wage rate is preference-free and independent of the underlying output price uncertainty under the efficiency wage hypothesis. Furthermore, we have shown that the introduction of the commodity futures market affects the firm's labor decision in general, and induces the firm to hire more labor and thereby produce more output if the firm is sufficiently risk averse in particular.

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