Why do agglomeration economies not raise employment?

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Abstract

There is ample evidence that the agglomeration of economic activities raises productivity. The available evidence, however, suggests that agglomeration economies do not appear to extend to helping raise employment in cities above less dense locales. But in the absence of other effects, standard economic reasoning suggests that more productive workers should also experience higher employment. This paper explains that the outward shift in the production function by improving investment incentives would also induce firms to create more specialized jobs. The specialized jobs are harder to fill and hence employment needs not go up, despite the increase in productivity.

Key words: job search and matching, agglomeration economies, unemployment.

JEL classifications: E24, J23, J64

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1. INTRODUCTION

There is ample empirical support that the agglomeration of economic activities in big cities raise productivity.[1] Standard economic reasoning suggests that if workers in big cities are made more productive by the agglomeration economies, employment should increase or equivalently unemployment should decline. The point is most easily seen in a model of a frictionless labor market with an upward sloping labor supply curve, where the increase in productivity shifts out the labor demand curve, raising wages as well as employment. The same conclusion applies equally well in a model of a frictional labor market, with the matching of workers and job vacancies governed by some constant returns to scale job matching function. To see this, let f be the production function, v and u the numbers of job vacancy and unemployed workers respectively, $m(v, u)$ a constant returns job matching function and c the cost of creating a job vacancy. In equilibrium, free entry equates the expected payoff to job creation to the cost of posting a job vacancy:

$$m(1, q)(f - w) = rc. \tag{1}$$

where $q = u/v$ and w and r are wages and the interest rate respectively. With labor market frictions, any productivity increase that results in an outward shift of the production function f will not in general be entirely dissipated in higher wages, but will rather be shared between the firm and the worker in a job match. The remaining channel for the higher payoff to job creation to be competed away is via more job creation, lowering the probability that each vacancy will be filled. Common to both models is that the productivity gains are partially dissipated in higher wages and partially in increased employment.

The available evidence, however, suggests that density has little systematic influence on unemployment. The study by Alperovich (1993) did find that larger cities

[1] For more recent evidence, see Rauch (1993) and Ciccone and Hall (1996).

tend to have lower unemployment in a sample of Isaeli cities. But in two early studies, Vipond (1974) and Sirmans (1977) uncover positive relationships between the rate of unemployment and city size in a sample of British cities. More recently, Maillard (1997) finds an inverted–U relationship between unemployment and city size among French cities, where unemployment is increasing in city size up to cities with a population of one million. Furthermore, Glaeser et al. (1995) find that the raw correlation between unemployment and population is essentially zero in a sample of 203 US cities. Though they did not report the conditional correlations, their discussion

> "...Southern cities in the sample tend to have relatively smaller populations, ...dramatically lower per capita incomes, lower unemployment... Northeastern cities are large, have...higher per capita income, higher unemployment..."

suggests that the conditional correlations between unemployment and city size may well be positive.

What might explain the absence of a negative relationship between unemployment on the one hand and city size and productivity on the other hand?

In a small town, jobs are usually less capital intensive as well as less specialized. An average worker would qualify quite well for many different types of jobs.[2] On the other hand, jobs in cities are usually more capital intensive and more specialized. A typical job vacancy may only find a good match in workers that possess some specific skills. In this way, there may not be greater job creation in big cities or the increased job creation may not result in higher employment if the big city jobs are more specialized

[2] In the movie Local Hero, the innkeeper in the small Scottish seaside town where the oil company executive Burt Lancaster plans to build an oil refinery doubles as the mayor as well as the town's only certified accountant. While it is difficult to verify how representative this folklore is, it is probably fair to say that it is far from being a figment of the scriptwriter's imagination.

and so harder to fill. These are certainly not novel ideas.[3] The task is to construct an equilibrium theory of the labor market in which firms would be inclined to create more specialized jobs in a denser labor market, and more importantly to establish that this would indeed give rise to the possibility that employment may fall in the midst of rising productivity.

The labor market model I study in this paper is based on the wage–posting model first proposed by Peters (1991) and Montgomery (1991) in which search frictions arise because of the lack of coordination in workers' applications to job vacancies. To it, I add heterogeneous job matches with which the output of a job match depends continuously on the *quality* of the match. The main element in the model is the tradeoff the firm faces in choosing between more capital intensive and specialized jobs and less capital intensive ones. In the absence of match imperfections, more capital intensive jobs are more productive. But these jobs are also more exact in their skill requirements and less tolerant of any match imperfections. Indeed, these jobs may yield very little output if not matched to workers with the appropriate skills.

If agglomeration economies and capital investment are complementary, the outward shift of the production function due to the productivity gains from increasing density would be followed by increases in capital investment. Employment may decline in the mean time, however, because the more capital intensive and specialized jobs created are harder to fill. Indeed in some cases, there may in fact be fewer jobs created in equilibrium. This happens because with search frictions, the more capital intensive jobs created are only viable when each attracts a greater number of applicants on average. With few applicants to choose from, firms are unlikely to find any acceptable matches for the now more specialized vacancies to make good use of the more capital intensive technology. In this way, the productivity gains may not be dissipated in more job creation and higher employment but in the creation of jobs that are harder

[3] See Hall (1989).

to fill. Nevertheless, aggregate output will increase because of the productivity gains from greater density, as well as from the creation of more capital intensive jobs.

While the channel through which agglomeration economies operate is still an area of active research, two leading hypotheses have emerged. The first one, initially proposed by Alfred Marshall and elaborated in Jacobs (1968) and Lucas (1988), is the human capital externality that arises through the day to day interactions among workers locating in close proximity. The second one is the returns to market size in the presence of fixed costs, which has been the workhorse in recent research in economic geography.[4] While both explanations appear equalling compelling, I shall assume that the agglomeration economies arise out of the returns to market size given rise by the fixed cost in production in this paper. The model is a richer model, with the productivity of workers, product variety and the job queue all endogenously determined and interdependent.

On a more general level, the theory in this paper is a theory of why unemployment exhibits no downward trend as productivity increases over time. The discussion in Pissarides (2000, chapter 1) suggests that the easiest explanation is to argue that the cost of creating a job vacancy is straightly proportional to wages. In (1), if w and c rise in proportion to productivity f, the job queue and therefore unemployment is not affected by the level of productivity. The difficulty with this explanation is that unless job creation is 100 percent labor intensive, the cost of creating a job vacancy should rise less than proportionately than wages. In practice, job creation is often not 100 percent labor intensive for physical capital usually does play some role in setting up the job vacancies and in recruiting qualified workers. In the context of why unemployment should not be systematically lower in big cities, the argument seems even less compelling. For the large firms in big cities often adopt more capital intensive technologies and can exploit the economies of scale better in the recruitment

[4]See Masahisa, Krugman and Venables (1999) for a most updated survey.

process.

A further possible objection to the analysis is that in reality, the sheer size of the big city and the more specialized human capital investment in large metropolitan areas would mitigate the job–matching problems in the big city associated with the jobs being more specialized to a significant extent. Firms in the city of London employ a large number of people in very specialized jobs. Undoubtedly, these jobs are harder to fill for firms and harder to find for workers. Yet in a labor market as large as London's, the probability for the same specialized vacancy to find a good match should be higher than in less dense areas. That may be the case as there would be a far greater number of workers in different specializations available in London simply because of her sheer size. Furthermore, the greater specialization of jobs in London would also induce her labor force to specialize to a greater extent in human capital investment.

The two forces would certainly help resolve the job–matching problems in the big city somewhat. But unless technologies and the skill requirements of jobs are unchanging, it is unlikely that they would completely undo the effects of specialization on job-matching described in this paper. Indeed, in the steady state of an economy where technologies and the skill requirements are stationary, job–matching problems would be minimal no matter how specialized the jobs are. In practice, it is technological change that is primarily responsible for matching frictions. True, over time, the sorting of more specialized workers into big cities and the more specialized human capital investment may mitigate the matching problems to a certain extent. But so long as technologies and the skill requirements of jobs are continuously changing, the sorting and the human capital investment would never completely catch up.[5] Finding

[5] A good case in point is the need for hi–tech firms in the Silicon Valley to import engineers from around the world, apparently given rise by human capital investment in the US not catching up entirely with the technological change.

the right match for the more specialized jobs in the big city should thus still be harder than finding the right match for the less specialized jobs in less dense areas.

The paper is closely related to Acemoglu and Shimer (1999b, 2000) who also entertain the notion that more capital intensive jobs are more specialized jobs that are harder to fill in their study on how the provision of unemployment insurance may raise output. The next section presents the model and contains the main result of the paper that employment needs not go up while workers are made more productive by agglomeration economies. The analysis of socially optimum capital investment and job creation follows in section 3. Section 4 provides some further defence to the hypothesis that the creation of more specialized jobs should make job–matching a more intricate problem. All proofs that do not follow directly from the discussion are relegated to the appendix. I shall restrict the analysis in this paper to a two–period version of the model for ease of exposition. The generalization to an infinite horizon economy is available in the working paper version of this paper.[6]

2. MODEL

The city is a closed economy populated by a continuum of workers of measure H. In the first period, firms enter, invest in physical capital, post jobs and announce wages. In the second period, workers apply to jobs after which job matches are formed. Then production commences, wages are paid and the city is closed down.

A. Monopolistic competition

There is a total of N (to be determined endogonously) monopolistically competitive firms in the city, each of which produces a differentiated intermediate good. Let y_i be the quantity of the ith intermediate good. The production function for the final

[6] Available at http://www.econ.hku.hk/˜tsechung/index.htm

good is

$$y = \left(\int_0^N y_i^{\lambda} di \right)^{\frac{1}{\lambda}}. \tag{2}$$

With the final good sector perfectly competitive and the price normalized to one, each intermediate good firm faces the (inverse) demand curve:

$$p_i = y^{1-\lambda} y_i^{\lambda-1}.$$

There is a fixed cost equal to c_e which the firms will incur at the time of entry.

B. Technology

Each monopolistically competitive firm $i \in [0, N]$ may post some J_i jobs, each of which may be matched with one and only one worker. If k is the capital installed for the job, the output of a worker–job match is given by

$$x = k(1 - \alpha\theta).$$

with the cost of capital investment equal to $c_k(k) = k^\gamma$ for some $\gamma > 1$. Each firm uses a product–specific technology that would be most productive when matched with workers who possess some specific skills. The productivity of a worker–firm match is therefore idiosyncratic. The variable θ is to measure the quality of a worker–firm match with $\theta = 0$ a perfect match. And so the ideal output of the job is equal to $x = k$, with x falling below the ideal when the match is less than perfect. For simplicity, I assume that θ is uniformly distributed on the unit interval and is iid across any possible worker–firm pairs. The exact value of θ for a given match is not known until the firm and worker make contact so that it is not possible for firms and workers to direct their searches toward the best possible match.[7]

[7] In princple, the results to follow should still hold if searches are directed so long as some imperfections remain in the knowledge of θ before the pair makes contact.

The percentage output loss due to a given worker-firm mismatch is governed by the parameter α. More capital intensive jobs are usually jobs that use more advanced technologies. These jobs are typically more exact in their skill requirements. Hence, the output loss due to a given worker–firm mismatch would be greater for more capital intensive jobs, from which it follows that α should be a positive function of k. For analytical convenience I shall work with the following parameterization:

(A1) $\quad \alpha = k^{\frac{1}{\phi}} + 1$

for some $\phi > 0$. The addition of 1 to $k^{\frac{1}{\phi}}$ ensures that $\alpha > 1$ in the firm's optimum, which as will be seen in the following is necessary to guarantee an interior solution to its profit maximization. I shall further assume that the returns to variety parameter in the production of the final good

(A2) $\lambda \in \left(\frac{1}{2}, 1\right)$

which serves to ensure that the social optimum is well defined.[8]

C. Search frictions

Workers make their job applications after firms make capital investment, post vacancies and announce wages. The matching of workers and job vacancies follow the wage–posting model of Montgomery (1991) and Peters (1991) where each worker may apply to no more than a single job opening. In the absence of coordination among workers in their job applications, a job opening may thus attract none, one or more than one applicants. Conversely a given worker may not land a job if he happens to apply to a vacancy that attracts at least one other applicants.

The present analysis differs from the existing models for firms may find it optimum

[8] In the absence of search frictions and without capital investment, aggregate output net of the costs of entry is equal to $y = HN^{\frac{1}{\lambda}-1}\overline{x} - c_e N$ where $\overline{x}$ is average output of intermediate good per worker. Optimum N exists only when $\frac{1}{\lambda} - 1 < 1$ or $\lambda > \frac{1}{2}$. It will be seen in the following that the same restriction applies in the presence of search frictions and capital investment.

not to hire for a given vacancy if none of the applicants are deemed to be an acceptable match. In particular, the firm may set a minimum match requirement θ_c and not hire at all for the vacancy if not a single applicant has a match parameter $\theta \leq \theta_c$.

Suppose a given vacancy has received n applications. The best match among the n applicants would then be some $\theta_n = \min_{j=1,\ldots,n} \{\theta_j\}$ that has a CDF given by $G_n(\theta_n) = 1 - (1 - \theta_n)^n$ since each θ_j is uniformly distributed on the unit interval. The probability that the vacancy would successfully recruit is therefore equal to $1 - (1 - \theta_c)^n$ and the expected output that the vacancy yields is given by

$$x_n = (1 - (1 - \theta_c)^n)\, k \left(1 - \alpha E[\theta_n \mid \theta_n \leq \theta_c]\right) \tag{3}$$

where

$$E[\theta_n \mid \theta_n \leq \theta_c] = \frac{(1+n)^{-1}\left(1 - (1 - \theta_c)^{1+n}\right) - (1 - \theta_c)^n \theta_c}{1 - (1 - \theta_c)^n} \tag{4}$$

is the conditional expectation of θ_n given that $\theta_n \leq \theta_c$. Suppose each of some h workers apply to the given vacancy with probability p, the expected output the vacancy may eventually generate is therefore

$$\overline{x} = \sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} x_n \tag{5}$$

and the probability that the vacancy will successfully recruit is

$$\eta = \sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} \left(1 - (1 - \theta_c)^n\right).$$

If each of these h workers apply to each of some J vacancies with equal probability, then $p = 1/J$. Let $q = h/J$ be the queue of job applicants a given vacancy will attract on average and when h becomes large it can be shown that

Lemma 1

(a) $\overline{x}(q, \theta_c, k, \alpha) = k\left(1 - e^{-q\theta_c}\left(1 - \alpha\theta_c - \frac{\alpha}{q}\right) - \frac{\alpha}{q}\right)$,

(b) $\eta(q, \theta_c) = 1 - e^{-q\theta_c}$.

I have in the above assumed that each vacancy posted by a given firm recruits separately. This treatment does entail some loss of generality. For example suppose the firm posts two vacancies and one attracts two good matches and the other none. Then the above calculation assumes that only one vacancy would be filled. However if the firm can coordinate the recruitments among its vacancies, then both vacancies should be filled in this case.[9] The matching technology of the wage–posting model thus exhibits a certain increasing returns to scale at the firm level. I choose not to deal with this complication for the model quickly becomes intractable in doing so.

D. Profit maximization and equilibrium in wage posting

Although each vacancy that the firm posts may only successfully recruit with a probability less than one and the output yielded by each filled job is uncertain, in posting a continuum of job vacancies, these uncertainties vanish at the firm level. The firm will recruit for certain $J_i\eta\left(q,\theta_c\right)$ workers and produce $J_i\overline{x}\left(q,\theta_c,k,\alpha\right)$ units of output. For a given average queue length, a posted wage, a capital intensity and a minimum match requirement, the ith monopolistically competitive firm chooses the number of vacancies to post to maximize[10]

$$\pi_i\left(q,\theta_c,w,k,\alpha\right)=\max_{J_i}\left\{y^{1-\lambda}\left(J_i\overline{x}\left(q,\theta_c,k,\alpha\right)\right)^{\lambda}-J_i\left(w\eta\left(q,\theta_c\right)+k^{\gamma}\right)\right\}.$$

[9]This type of increasing returns is first noted by Arrow (1971) in his famous example where the expected output of a firm with 2 workers and 4 machines would be more than two times that of a firm with 1 worker and 2 machines where each machine breaks down with some exogenous probability.

[10]If the firm knows for certain that it may only fill $J_i\eta$ jobs, then it can be argued that it should install capital only for that many jobs, but not for all posted vacancies. However, under the assumption that the individual vacancies recruit separately without coordination, it is more appropriate to assume the firm has to install capital for each posted vacancy.

This results in

$$J_i(q,\theta_c,w,\alpha) = y\left(\frac{\lambda\overline{x}(q,\theta_c,\alpha)^{\lambda}}{w\eta(q,\theta_c) + (\alpha^{\phi}-1)^{\gamma}}\right)^{\frac{1}{1-\lambda}}, \tag{6}$$

$$\pi_i(q,\theta_c,w,\alpha) = y\left(\frac{\lambda\overline{x}(q,\theta_c,\alpha)}{w\eta(q,\theta_c) + (\alpha^{\phi}-1)^{\gamma}}\right)^{\frac{\lambda}{1-\lambda}}(1-\lambda). \tag{7}$$

where I have used (A1) that $k = \alpha^{\phi} - 1$ to substitute out k.

Let $\mu(q,\theta_c)$ equal to the probability that a worker will successfully obtain a job in applying to firms whose vacancies attract an average queue of q job applicants and which only accept matches with $\theta \leq \theta_c$. Assuming risk-neutral workers, the worker's expected utility is

$$u(q,\theta_c,w) = \mu(q,\theta_c)w. \tag{8}$$

Any triples of (q,θ_c,w) offered in equilibrium must yield workers the same expected utility for any triples that offer less attract no applicants. Let u^* be the level of utility workers obtain in equilibrium. Then any triples of (q,θ_c,w) offered in equilibrium must satisfy:

$$w = \frac{u^*}{\mu(q,\theta_c)}. \tag{9}$$

Next we can establish that

Lemma 2 $\mu(q,\theta_c) = \eta(q,\theta_c)/q$.

By lemma 2, $(7)-(9)$, we may state the firm's profit maximization as

$$\pi_i(u^*) = \max_{\{\alpha,q,\theta_c\}}\left\{y\left(\frac{\lambda\overline{x}(q,\theta_c,\alpha)}{u^*q + (\alpha^{\phi}-1)^{\gamma}}\right)^{\frac{\lambda}{1-\lambda}}(1-\lambda)\right\} \tag{10}$$

where the choice variables are technology α, queue length q and minimum match requirement θ_c. It may appear erroneous that the queue length could be a choice variable for the firm instead of an object to be determined in equilibrium. But by virtue of (9) , this is merely equivalent to stating the firm's maximization in terms of choosing a wage and a match requirement. And the calculations to follow are simpler when the maximization is stated in terms of q instead of w.

It follows immediately that optimal

$$\theta_c^* = \max_{\theta_c} \{\overline{x}(q, \theta_c, \alpha)\}$$

and from lemma 1a, we have $\theta_c^* = \alpha^{-1}$. Hence in equilibrium, the firm would hire as long as the best match for a vacancy yields positive output.[11, 12] The practical importance of this result is that it confirms that in choosing a more capital intensive technology, the firm not surprisingly will choose to be more selective in its hiring.

[11]Remember that θ is assumed bounded within the unit interval. Hence if $\alpha < 1$, the optimum hiring requirement θ_c^* is at the corner of 1. Assumption A1, by forcing $\alpha > 1$, ensures an interior solution.

[12]If wages are determined by bargaining and if the market for the intermediate good is competitive, that $\theta_c^* = \alpha^{-1}$ is obvious as the firm would certainly like to form the match as long as it yields positive output. That the same result also holds in the wage-posting model with monopolistic competition is less obvious. As the discussion above suggests, it stems from the competition in the labor market, along with the assumption that the matching technology is constant returns. It should perhaps be noted that the optimal recruitment policy is not time consistent. If the best match indeed has $\theta = \alpha^{-1}$, this worker may only produce zero output while the firm has commited to pay him a positive wage. In this case, the firm is better off not hire ex-post if it has the option to do so. The time inconsistency is not surprising for the firm in choosing its hiring policy has its goal the maximization of *expected* payoff but not avoiding negative payoff as such. In raising the hiring requirement to be time consistent, it is necessary to pay a higher wage to compensate workers for the falling matching probability or face the consequence of attracting of a shorter job queue. Either way, the firm's expected payoff must only fall. For simplicity, I choose not to deal with the time inconsistency and assume that the firm can commit to the announced recruitment policy. The main result of the paper should survive imposing time consistency while doing so complicates the analysis a great deal more.

Substitute θ_c^* back to lemma 1a, $\overline{x}$ is simplified to

$$\overline{x}(q,\alpha) = \left(\alpha^{\phi} - 1\right)\left(1 - \frac{\alpha}{q}\left(1 - e^{-q/\alpha}\right)\right). \tag{11}$$

and η becomes

$$\eta\left(\frac{q}{\alpha}\right) = 1 - e^{-q/\alpha}. \tag{12}$$

In addition to $\theta_c^* = \alpha^{-1}$, we now have from the maximization in (10), as functions of u^*, the firm's optimal α and q which together imply a wage offer from (9). The final step is to solve for u^* via a free entry condition that firms in the intermediate good sector earn zero profit in equilibrium:

$$\pi_i(u^*) = c_e.$$

Evidently, this procedure is equivalent to solving[13]

$$u^* = \max_{\{q,\alpha,w\}}\left\{\frac{\eta(q/\alpha)}{q}w\right\} \tag{13}$$

subject to the firms earning zero profit:

$$y\left(\frac{\lambda\overline{x}(q,\alpha)}{w\eta\left(\frac{q}{\alpha}\right) + \left(\alpha^{\phi} - 1\right)^{\gamma}}\right)^{\frac{\lambda}{1-\lambda}}(1-\lambda) = c_e. \tag{14}$$

Solving for ηw in the above and substitute the result back into (13), the problem simplifies to

$$u^* = \max_{\{q,\alpha\}}\left\{\lambda\left(\frac{(1-\lambda)y}{c^e}\right)^{\frac{1-\lambda}{\lambda}}\frac{\overline{x}(q,\alpha)}{q} - \frac{\left(\alpha^{\phi}-1\right)^{\gamma}}{q}\right\} \tag{15}$$

In the above, the two terms $\overline{x}/q$ and $\left(\alpha^{\phi}-1\right)^{\gamma}/q$ denote respectively the expected output of intermediate good per worker and capital investment per worker.[14] That

[13]This equivalence is discussed in details in Acemoglu and Shimer (1999a,b) and Moen (1997).

[14]Since all firms offer the same wage and set the same minimum hiring requirement in equilibrium, q is the economywide average job queue which is equal to H/NJ_i. Then $\overline{x}/q = \overline{x}NJ_i/H$. With $\overline{x}$ equal to the average output of intermediate good yielded by each job vacancy, $\overline{x}NJ_i$ is the aggregate output of intermediate good and $\overline{x}NJ_i/H$ the per capita output. Since $\alpha^{\phi} - 1 = k$, $\left(\alpha^{\phi}-1\right)^{\gamma}/q = k^{\gamma}NJ_i/H$ is per capita capital investment.

the first term is augmented by the factor $\lambda\left((1-\lambda)\,y/c_e\right)^{\frac{1-\lambda}{\lambda}}$ reflects the fact that the marginal revenue of a unit of intermediate good to the monopolistically competitive firms depends on aggregate output, the fixed cost of entry as well as the price elasticity of demand these firms face. The first order conditions for q and α are respectively

$$\lambda\left(\frac{(1-\lambda)\,y}{c^e}\right)^{\frac{1-\lambda}{\lambda}}\frac{\partial\left[\overline{x}\left(q,\alpha\right)/q\right]}{\partial q}-\frac{\partial\left[\left(\alpha^{\phi}-1\right)^{\gamma}/q\right]}{\partial q}=0, \tag{16}$$

$$\lambda\left(\frac{(1-\lambda)\,y}{c^e}\right)^{\frac{1-\lambda}{\lambda}}\frac{\partial\left[\overline{x}\left(q,\alpha\right)/q\right]}{\partial \alpha}-\frac{\partial\left[\left(\alpha^{\phi}-1\right)^{\gamma}/q\right]}{\partial \alpha}=0. \tag{17}$$

The solution of the equilibrium is yet to be completed since aggregate output y is yet to be specified. To do so, I now turn to the general equilibrium.

E. The general equilibrium

Let $\widehat{x}$ be the average output of intermediate good yielded by a filled vacancy, then by definition $\overline{x}=\widehat{x}\eta$ from which it follows

$$\widehat{x}=\frac{\overline{x}}{\eta}.$$

And this must clearly equal to the average output of intermediate good an employed worker produces. With each worker employed with probability $\mu=\eta/q$, total employment is equal to

$$E=H\frac{\eta}{q}.$$

From (2), aggregate output may be expressed as

$$y=N^{\frac{1}{\lambda}}\frac{H}{N}\frac{\overline{x}}{q} \tag{18}$$

In equilibrium, the job queue must satisfy $q=H/NJ_i$ and using (6) it becomes

$$q=\frac{H}{Ny}\left(\frac{\lambda\overline{x}^{\lambda}}{w\eta+\left(\alpha^{\phi}-1\right)^{\gamma}}\right)^{\frac{1}{\lambda-1}}. \tag{19}$$

Equations (18), (19) and the zero profit condition (14) imply

$$y = \frac{N}{1-\lambda} c_e, \tag{20}$$

$$N = \left(\frac{H(1-\lambda)}{c_e} \frac{\overline{x}}{q} \right)^{\frac{\lambda}{2\lambda-1}}. \tag{21}$$

The first equation states that aggregate output is increasing in product variety while the second one states that for a given output of intermediate good per worker, product variety rises with market size H normalized by the cost of entry c_e. The latter relationship captures the well–known increasing returns to scale associated with models with monopolistic competition and fixed costs of entry. The equilibrium is now completely specified by (20), (21), (16) and (17).

G. Comparative statics

To check how increases in market size H affect productivity, technology choice and employment, first apply (20) and (21) to (16) and simplify

$$\lambda \left(\frac{H(1-\lambda)}{c_e} \left(1 - \frac{1-e^{-\widehat{q}}}{\widehat{q}} \right) \right)^{\frac{1-\lambda}{2\lambda-1}} \left(2\frac{1-e^{-\widehat{q}}}{\widehat{q}} - e^{-\widehat{q}} - 1 \right) + \left(\frac{1}{\widehat{q}^{\phi}} - \frac{1}{q^{\phi}} \right)^{\gamma - \frac{\lambda}{2\lambda-1}} q^{\phi\gamma + \frac{1-\lambda(1+\phi)}{2\lambda-1}} = 0 \tag{22}$$

where $\widehat{q} = q/\alpha$. The above is the condition which pins down the equilibrium job queue for a given α. I shall refer to the above as the market tightness condition in the following. Under the assumption[15]

(A3) $\gamma - \dfrac{\lambda}{2\lambda-1} \geq 0,$

Lemma 3

[15]If (A3) is violated, (22) often has no solution. The assumption thus helps ensure the existence of equilibrium as well as determine the shape of the relationship defined by (22).

(a) The left hand side of (22) is decreasing in $\widehat{q}$ and H and increasing in q.

(b) Equation (22) defines a positive-valued implicit function $q = F(\widehat{q}) > \widehat{q}$ for $\widehat{q} > 0$,

(c) $dF(\widehat{q})/d\widehat{q} > 0$,

(d) $\lim_{\widehat{q} \to 0} F(\widehat{q}) = 0$ and $F(\widehat{q})$ unbounded.

If α were fixed, the equilibrium is entirely determined by (22) as an equation in q, with $\widehat{q}$ in the equation replaced by its definition q/α. The model behaves like a standard job–matching model of unemployment in which productivity increases would be followed by greater employment. In particular, increases in market size H would in the present case, through raising the productivity of the job vacancies, raise employment in equilibrium. To see this, note that as H increases, it follows from lemma 3(i) that $\widehat{q}$ would fall. For a given α, the decline in $\widehat{q}$ is merely the fall in q. Falling job queue is equivalently greater job creation. In the absence of the technology choice, there can be no changes in firms' match requirement and employment must rise as a result.

In the presence of the technology choice, the conclusion can be turned around completely. To proceed, I next combine (16) and (17) to yield

$$q = \widehat{q}\left(1 + \phi \frac{\widehat{q}\left(\gamma\left(1 + e^{-\widehat{q}}\right) - 1\right) + \left(1 - e^{-\widehat{q}}\right)(1 - 2\gamma)}{1 - e^{-\widehat{q}}(1 + \widehat{q})}\right)^{-\frac{1}{\phi}}. \tag{23}$$

I will call the above the technology choice condition as the condition embeds the equilibrium condition for α. For $\phi > 1$, the expression inside the big bracket turns negative for small values of $\widehat{q}$ and so equilibrium may fail to exist. I shall then restrict attention to the case of

(A4) $0 < \phi \leq 1$.

Lemma 4 In (23),

(a) If $\phi = 1$,

(i) q is decreasing in $\widehat{q}$ for $\widehat{q} > 0$,

(ii) $\lim_{\widehat{q}\to 0} q = 3(\gamma - 1)^{-1}$ and $\lim_{\widehat{q}\to\infty} q = (\gamma - 1)^{-1}$.

(b) If $\phi < 1$,

(i) q is increasing in $\widehat{q}$ for $\widehat{q}$ sufficiently close to 0,

(ii) $\lim_{\widehat{q}\to 0} q = 0$ and $\lim_{\widehat{q}\to\infty} q = 0$.

A direct corollary of part (i) above is that

Corollary If $\phi = 1$, in equilibrium, an increase in α is followed by an increase in q.

The corollary may be understood by the fact that a longer job queue may be necessary to sustain the optimality of more capital intensive and specialized jobs as these jobs are less forgiving in match imperfections and so harder to fill. However that is the case only if $\phi = 1$. For $\phi < 1$, an increase in α can be followed by a decline in the job queue. In this case, the increased profitability of job creation may more than compensate for the increased difficulty of finding the right matches for the more capital intensive jobs at the old level of the job queue.

To formally analyze the comparative statics of market size on employment, I first turn to the case of $\phi = 1$. In this case, the technology choice condition is downward sloping throughout. Figure 1 depicts q as functions of $\widehat{q}$ from the technology choice and market tightness conditions. The equilibrium is at the intersection of the two conditions.

Now consider an increase in market size H. The technology choice condition is independent of H. As to the market tightness condition, it follows from lemma 3(a) that the condition would shift in as depicted in figure 2. In equilibrium, $\widehat{q}$ falls while q rises. The increase in q is equivalently a fall in job creation per worker. With

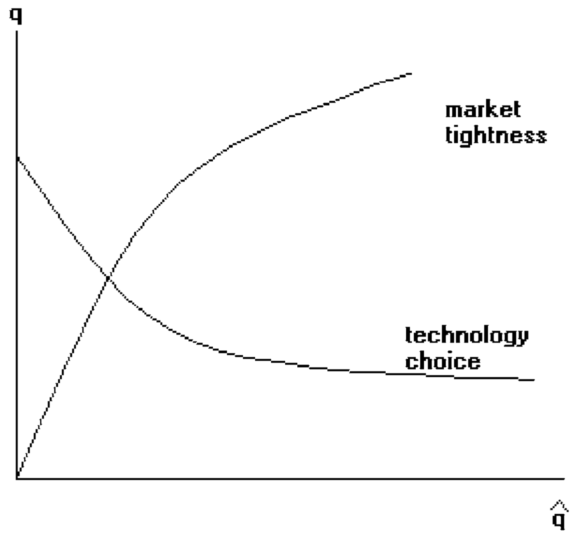

FIG. 1. The determination of equilibrium

$\widehat{q} = q/\alpha$ fallen but q increased, α must have gone up in the mean time. And indeed this explains why the productivity gains from the increase in market size are not dissipated in greater job creation since in this case the more capital intensive jobs created need a longer job queue to be viable. In all, equilibrium employment would decline since not only fewer jobs are created for each worker but also the jobs created are more exact in match requirement and so harder to fill.

Proposition 1 If $\phi = 1$, in equilibrium, job creation and employment decline as market size increases

The decline in employment, however, is not so much a perverse outcome because

Proposition 2 Entry (product variety), the capital intensity of the job vacancy, per

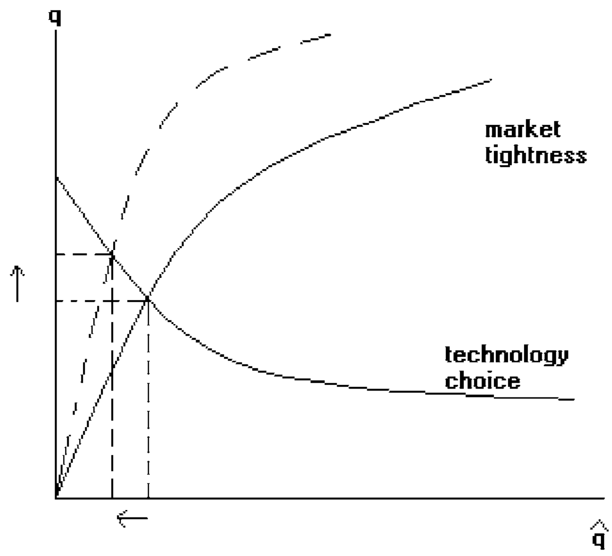

FIG. 2. Comparative statics of market size: rising job queue

capita output, wages and workers' expected utility all rise as market size increases.

The normal channels whereby increases in market size raise product variety and then the marginal revenue to the monopolistically competitive firms, capital investment and eventually aggregate output are still operative. It is now just that when the firms choose to invest in more capital intensive jobs, they would choose to offer fewer of them in equilibrium. Nevertheless, the effects of the increase in capital intensity and the agglomeration economies on output suffice to dominate the effect of the decline in employment.

The role of the technology choice in driving a possible negative relationship between market size and employment deserves further attention. For a given q, the more capital intensive jobs created would only recruit with a lower probability and there would also be greater penalty for a given mismatch. A longer job queue serves

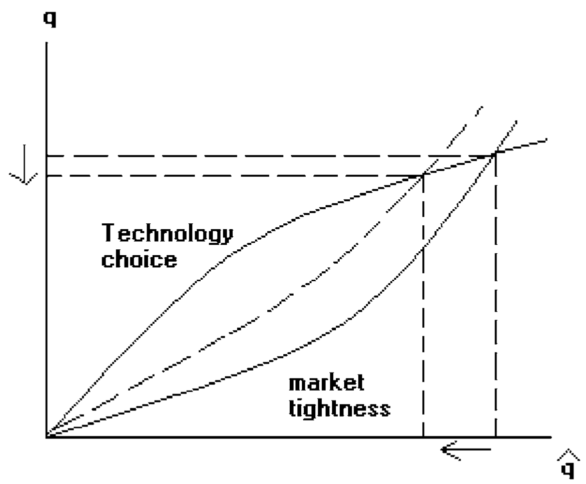

FIG. 3. Comparative statics of market size: falling job queue

to compensate firms for the falling hiring probability and helps lower the expected mismatch. If $\phi = 1$, the effect of the technology choice on the job queue will dominate the effect of the productivity gains due to greater product variety given rise by the increase in market size. The increase in the profitability of job creation is dissipated not in terms of more jobs offered but in terms of the jobs becoming less likely to find good matches for a given queue length. In particular, even though the job queue has lengthened, the hiring probability $\eta = 1 - e^{-\widehat{q}}$ which only depends on $\widehat{q}$ instead of increasing would go down with $\widehat{q}$ falling with H. With or without the technology choice, the jobs may only be filled with lower probabilities in equilibrium when job creation is now a more profitable business.

The empirical evidence points to an absence of a systematic relationship between market size and productivity on the one hand and employment on the other hand, not a negative relationship. Indeed the model does not point to a definite negative relationship either. For $\phi < 1$, the technology choice condition is initially increasing. And if the equilibrium occurs at where the technology choice condition cuts the market tightness condition from above as in figure 3, an increase in market size could conceivably raise employment. In this case, as the market tightness condition shifts up due to an increase in H, $\widehat{q}$ falls, lowering the probability that each job vacancy will be filled as in the case of $\phi = 1$. But now q declines in equilibrium which is equivalently an increase in job creation. The increase in job creation may certainly raise employment if its effect on employment more than offsets the effect of the falling matching probability due to the jobs becoming harder to find suitable matches.

3. SOCIAL OPTIMUM

The papers by Moen (1997) and Acemoglu and Shimer (1999a) have shown that in the absence of other distortions, both job creation and capital investment in the wage–posting model satisfy constrained optimality. Quite obviously, in the presence of monopolistic competition, constrained optimality should not survive in the present model. The more interesting question is in what way the distortions introduced by market power and the inefficient entry of firms in monopolistic competition affect labor market outcomes.

Given the search frictions and by virtue of (18), aggregate output net of capital investment and the cost of entry is

$$S = \max_{N,\alpha,q,\theta_c} \left\{ N^{\frac{1-\lambda}{\lambda}} H \frac{\overline{x}\left(q,\theta_c,\alpha\right)}{q} - H \frac{(\alpha-1)^{\gamma}}{q} - c_e N \right\}. \tag{24}$$

First the optimal minimum match requirement satisfies

$$\max_{\theta_c} \left\{ \overline{x}\left(q,\theta_c,\alpha\right) \right\}$$

yielding $\theta_c = \alpha^{-1}$ as in equilibrium. Next, the first order condition with respect to

N yields[16]

$$N=\left(\frac{(1-\lambda)\,H}{c_e\lambda}\frac{\overline{x}\,(q,\alpha)}{q}\right)^{\frac{\lambda}{2\lambda-1}}. \tag{25}$$

For a given $\overline{x}/q$, on comparing (21) and the above, it can be seen that equilibrium entry is suboptimal.. This is hardly surprising as the monopolistically competitive firms fail to appropriate the entire surpluses that flow from their entries.

And then after substituting in N from (25), the first order conditions for q and α are respectively

$$\lambda^{\frac{1-\lambda}{1-2\lambda}}\left(\frac{(1-\lambda)\,H}{c_e}\frac{\overline{x}\,(q,\alpha)}{q}\right)^{\frac{1-\lambda}{2\lambda-1}}\frac{\partial\left[\overline{x}\,(q,\alpha)\,/q\right]}{\partial q}-\frac{\partial\left[(\alpha-1)^{\gamma}\,/q\right]}{\partial q}=0, \tag{26}$$

$$\lambda^{\frac{1-\lambda}{1-2\lambda}}\left(\frac{(1-\lambda)\,H}{c_e}\frac{\overline{x}\,(q,\alpha)}{q}\right)^{\frac{1-\lambda}{2\lambda-1}}\frac{\partial\left[\overline{x}\,(q,\alpha)\,/q\right]}{\partial \alpha}-\frac{\partial\left[(\alpha-1)^{\gamma}\,/q\right]}{\partial \alpha}=0. \tag{27}$$

The difference between the above and the corresponding equilibrium conditions in (16) and (17) after the substitution from (20) and (21) is that $\lambda^{\frac{1-\lambda}{1-2\lambda}}>1$ in the optimality conditions has taken the place of $\lambda<1$ in the equilibrium conditions.

By combining (26) and (27), it can be seen that the equilibrium technology choice condition (23) remains valid in the social optimum. But now in lieu of the market tightness condition (22) , we have under social optimality:

$$\lambda^{\frac{1-\lambda}{1-2\lambda}}\left(\frac{H\,(1-\lambda)}{c_e}\left(1-\frac{1-e^{-\widehat{q}}}{\widehat{q}}\right)\right)^{\frac{1-\lambda}{2\lambda-1}}\left(2\frac{1-e^{-\widehat{q}}}{\widehat{q}}-1-e^{-\widehat{q}}\right)+\\ \left(\frac{1}{\widehat{q}^{\phi}}-\frac{1}{q^{\phi}}\right)^{\gamma-\frac{\lambda}{2\lambda-1}}q^{\phi\gamma+\frac{1-\lambda(1+\phi)}{2\lambda-1}}=0 \tag{28}$$

whose only difference with (22) is that its first term is augmented by $\lambda^{\frac{1-\lambda}{1-2\lambda}}>1$ instead of by $\lambda<1$ in (22). Replacing the factor λ in the equilibrium condition (22)

[16]This verifies that the restriction in (A2) that $\lambda\geq 1/2$ is necessary for the social optimum to be well-defined.

by $\lambda^{\frac{1-\lambda}{1-2\lambda}} > \lambda$ clearly has the same qualitative effects on q and $\widehat{q}$ as those from an increase in H. Hence how q and $\widehat{q}$ and therefore α in equilibrium may deviate from optimality mimics the comparative statics of H.

Proposition 3

(a) In equilibrium, capital intensity and entry are suboptimal.

(b) If an increase in market size results in falling employment in equilibrium, then equilibrium employment is excessive. If an increase in market size results in rising employment, then equilibrium employment is suboptimal.

The inefficiency stems from two sources. First the suboptimal entry lowers the agglomeration economies. Second market power drives a wedge between the monopolistically competitive firm's marginal revenue and the shadow price of a unit of intermediate good. Either distortion lowers capital investment below the social optimum. Job creation however can be excessive. That may be the case since the more capital intensive jobs that should be created under the social optimum are less forgiving in match imperfections and may need a long job queue to sustain their optimality. The socially optimum job creation would indeed exceed equilibrium job creation if this effect dominates the tendency of suboptimal job creation in equilibrium arising from market power and the suboptimal agglomeration economies.

Although job creation and employment may be excessive in equilibrium,

Proposition 4 Per capita output is always suboptimal.

With or without the technology choice, monopolistic competition results in suboptimal production. Without the technology choice, the greater output may only be produced with more jobs created and more workers employed. With the technology choice and if $\phi = 1$, it would be optimal for the greater output to be produced via a smaller number of the more capital intensive jobs.

4. CONCLUDING REMARK

Perhaps the counter-argument that if the jobs created are more capital intensive and more specialized, more of them would be created in equilibrium, raising each worker's probability of finding a good match sounds equally plausible. Obviously, this argument would not hold if it is vacancies that attract workers in the job–matching process. Whereas if job–matching is characterized by workers attracting vacancies, the counter-argument could well be valid. But whatever the underlying reasons may be, the great majority of job matches in practice are formed by vacancies attracting workers.

APPENDIX

Proof of Lemma 1

Substitute (4) into (3) and then into (5)

$$\begin{aligned} X_n &= \sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} \times \\ & \quad k\left\{1-(1-\theta_c)^n - \alpha\left((1+n)^{-1} - (1+n)^{-1}(1-\theta_c)^{1+n} - (1-\theta_c)^n \theta_c\right)\right\}. \end{aligned} \tag{29}$$

Now evaluate the summations of each term inside the curly bracket separately:

$$\sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} (1-(1-\theta_c)^n) = 1-(1-p\theta_c)^h,$$

$$\sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} \frac{1}{1+n} = \frac{1-(1-p)^{h+1}}{p(h+1)} - (1-p)^h,$$

$$\sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} \frac{(1-\theta_c)^{1+n}}{1+n} = \frac{(1-p\theta_c)^{h+1}-(1-p)^{h+1}}{p(h+1)} - (1-\theta_c)(1-p)^h,$$

$$\sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} (1-\theta_c)^n \theta_c = \theta_c \left((1-p\theta_c)^h - (1-p)^h \right).$$

Combining the above into (29), let $p = \frac{1}{J}$ and $q = \frac{h}{J}$ and take limit as $h \to \infty$ yields part (a). Part (b) is simply the limit of the first line above when $h \to \infty$.

Proof of Lemma 2

Hosios (1990) shows that a worker that applies to a vacancy that attracts an average queue length of q would get hired with probability $\frac{1-e^{-q}}{q}$ if the firm randomly picks one applicant among the possibly many applicants that it has attracted. Lemma 1b shows that a vacancy that attracts an average queue length of q with a minimum match requirement of $\theta \leq \theta_c$ is as if it has an average queue length of $q\theta_c$ that hires any $\theta \leq 1$. Suppose the match between the given worker and the firm satisfies $\theta \leq \theta_c$. The worker then has an equal probability among the other qualified applicants to be the best match and ex-ante, it is as if the firm would pick the worker with equal probability as others. Therefore conditional on the worker qualifies as an acceptable match, he would obtain employment with probability $\frac{1-e^{-q\theta_c}}{q\theta_c}$. Since the worker qualifies as an acceptable match with probability θ_c, $\mu(q, \theta_c) = \eta(q, \theta_c)/q$.

Proof of Lemma 3

Under assumption (A3), simple algebra verifies that the left side of (22) is decreasing in $\widehat{q}$ and H and increasing in q. If the equation defines an implicit function for $q = F(\widehat{q})$, $F(\widehat{q})$ is an increasing function. Clearly the solution of q to (22) must exceed $\widehat{q}$. Then $F(\widehat{q})$ if it exists is positive-valued. The first term of (22) is negative and independent of q, while the second term is increasing in q without bound and tends to 0 as $q \to \widehat{q}$. This implies a unique solution of $q > \widehat{q}$ exists. As $\widehat{q} \to 0$, the first term of (22) vanishes and the second term may only vanish too if $q \to 0$. Finally, as $\widehat{q} \rightarrow \infty$, $\frac{1-e^{-\widehat{q}}}{\widehat{q}} \rightarrow 0$ so that the first term of (22) tends to a constant while the second term increases without bound for fixed q and so the solution for q too must increase without bound.

Proof of Lemma 4

The lemma can be established by simple differentiation and by applying the L'Hospita rule.

Proof of Corollary

Suppose an increase in α is followed by a decrease in q. If q decreases, lemma 4(a.i) implies that $\widehat{q}$ must have increased. A decrease in q and an increase in $\widehat{q} = q/\alpha$ imply that α must have risen, contrary to the hypothesis that α has gone up. This proves that an increase in α must be followed by a similar increase in q.

Proof of Proposition 2

I should first establish that capital intensity must rise in a larger market. We have seen how that is the case for $\phi = 1$ in the main text. To see that the same conclusion applies for $\phi < 1$, first note that (23) can be rewritten as

$$\alpha = \left(1 + \phi \frac{\widehat{q}\left(\gamma\left(1+e^{-\widehat{q}}\right)-1\right)+\left(1-e^{-\widehat{q}}\right)(1-2\gamma)}{1-e^{-\widehat{q}}(1+\widehat{q})}\right)^{-\frac{1}{\phi}}$$

which is always decreasing in $\widehat{q}$ for all $\phi \leq 1$. Now note that the agglomeration economies would raise both capital investment and the profitability of job creation. In equilibrium, the firm's recruitment probability $\eta = 1 - e^{-\widehat{q}}$ must then fall. This implies that $\widehat{q}$ is always decreasing in H and so the positive relationship between H and α follows. Next, the market tightness condition may be rewritten as

$$\lambda \left(\frac{H(1-\lambda)}{c_e}\frac{\overline{x}(q,\alpha)}{q}\right)^{\frac{1-\lambda}{2\lambda-1}} \left(2\frac{\alpha}{q}\left(1-e^{-q/\alpha}\right)-1-e^{-q/\alpha}\right)+\left(\alpha^{\phi}-1\right)^{\gamma} = 0. \quad (30)$$

Now that with $\widehat{q}$ fallen but α risen as H increases, the term $\lambda \left(\frac{H(1-\lambda)}{c_e} \frac{\overline{x}(q,\alpha)}{q} \right)^{\frac{1-\lambda}{2\lambda-1}}$ in the equation above must have increased since

$$\left(2\frac{\alpha}{q} \left(1 - e^{-q/\alpha} \right) - 1 - e^{-q/\alpha} \right) < 0$$

falls and $\left(\alpha^{\phi} - 1 \right)^{\gamma}$ in the same equation rises as they are decreasing in $\widehat{q}$ and increasing in α respectively. By virtue of (20) and (21), this implies that y and N are increasing in H. It follows from the maximization in (15) that this raises u^* and $\overline{x}/q$. Output per worker as given by

$$\frac{y}{H} = N^{\frac{1}{\lambda}-1} \frac{\overline{x}}{q} \tag{31}$$

must increase too as N and $\overline{x}/q$ have both risen. For $\phi = 1$, the increase in w follows trivially from the increase in expected utility being accompanied by the decline in the probability of employment. For $\phi < 1$, the probability of employment may rise and so w may decline even though expected utility is guaranteed to have risen. To see how w must also have increased in the midst of rising employment, note that the opportunity cost of letting a more capital intensive job vacant is higher. If w is indeed lowered in equilibrium, a deviating firm must be able to earn positive profit from posting a higher wage to attract a longer job queue.

Proof of Proposition 3

The conclusions on job creation q, capital intensity α and thus on employment $\mu = \left(1 - e^{-\widehat{q}} \right) / q$ are obvious from the discussion in the text. The conclusion on entry may be proved as follows. First denote values of the social optimum by the superscript o and equilibrium values by the superscript $*$. The proof of proposition 2 has shown that in equilibrium, $\partial \left(\overline{x}/q \right) / \partial H > 0$. Hence $\left(\overline{x}/q \right)^o > \left(\overline{x}/q \right)^*$. Equation (25) and (21) then imply $N^o > N^*$.

Proof of Proposition 4

By virtue of (31) , the proposition follows trivially since both $N^o > N^*$ and $(\overline{x}/q)^o > (\overline{x}/q)^*$.

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Not to be considered for publication

5. THE INFINITE HORIZON ECONOMY

I now generalize the analysis to an infinite horizon economy. Time is continuous and the interest rate is constant at some r. The matching of unemployed workers and vacant jobs take place at each moment of time. Unemployment in the steady state arises because a given worker–firm match is assumed to break down at some exogenous probability s. When the separation occurs, the worker returns to the unemployment pool and the capital installed for the job gets destroyed. All agents are risk neutral. Firms maximize expected discounted profit. Workers maximize expected discounted lifetime income. In this case, Acemoglu and Shimer (1999a) show that the discounted lifetime income of a currently unemployed worker is given by

$$u = \frac{\eta(q, \theta_c) w}{(r+s) q + \eta(q, \theta_c)} \tag{32}$$

in applying to job vacancies where his instantaneous probability of obtaining employment is $\mu = \eta(q, \theta_c)/q$. In equilibrium, unemployed workers must derive the same expected lifetime income from applying to any job vacancies. Then any triples of (q, θ_c, w) offered in equilibrium must satisfy

$$w = u^* \left((r+s) \frac{q}{\eta(q, \theta_c)} + 1 \right). \tag{33}$$

Let $J_i(t)$ be firm i's stock of unfilled vacancies and $E_i(t)$ its employment at time t. They satisfy

$$\dot{E}_i(t) = \eta(q, \theta_c) J_i(t) - sE_i(t). \tag{34}$$

The departure of $sE_i(t)$ workers at each t lowers the firm's productive capacity by $sE_i(t) \widehat{x}_i(t) = sy_i(t)$, while the recruitment of $\eta(q, \theta_c) J_i(t)$ workers would augment it by $\overline{X}(q,\theta_c,\alpha) J_i(t)$. This implies

$$\dot{y}_i(t) = \overline{X}(q,\theta_c,\alpha) J_i(t) - s y_i(t). \tag{35}$$

Denote the number of new vacancies created by the firm at time t by $j_i(t)$. The stock of unfilled vacancies evolves according to

$$\dot{J}_i(t) = j_i(t) - \eta(q,\theta_c) J_i(t). \tag{36}$$

At each t, the firm's cash flow is

$$\begin{aligned} \pi_i(t) &= y(t)^{1-\lambda} y_i(t)^{\lambda} - wE_i(t) - k(t)^{\gamma} j_i(t) \\ &= y(t)^{1-\lambda} y_i(t)^{\lambda} - wE_i(t) - (\alpha-1)^{\gamma} j_i(t). \end{aligned} \tag{37}$$

Taking aggregate output $y(t)$ as given, the firm maximizes discounted profit

$$V_i = \int_0^{\infty} e^{-rt} \pi_i(t)\, dt$$

subject to $(33)-(36)$. For simplicity, I should assume that $\phi = 1$ in the following.

Lemma 1 *In the steady state, the solution of the above optimal control problem is characterized by*

$$y_i = y\left(\frac{\lambda \overline{X}}{w\eta + (\alpha-1)^{\gamma}(\eta+r)(r+s)}\right)^{\frac{1}{1-\lambda}}. \tag{38}$$

$$(r+s)\eta\gamma(\alpha-1)^{\gamma-1} - \frac{\partial \overline{X}}{\partial \alpha}\frac{1}{\overline{X}}\left(w\eta + (\alpha-1)^{\gamma}(\eta+r)(r+s)\right) = 0, \tag{39}$$

$$\frac{\partial \eta}{\partial \theta_c}\left((\alpha-1)^{\gamma} - u^* \frac{r}{s}\frac{q}{\eta} + \frac{u^*}{r+s}\right) - \frac{\partial \overline{X}}{\partial \theta_c}\frac{1}{\overline{X}}\left(\frac{w\eta}{r+s} + (\alpha-1)^{\gamma}(\eta+r)\right) = 0, \tag{40}$$

$$\begin{aligned} &\frac{\partial \eta}{\partial q}\left((\alpha-1)^{\gamma} - u^* \frac{r}{s}\frac{q}{\eta} + \frac{u^*}{r+s} + u^* \frac{r+s}{s}\left(\frac{\partial \eta}{\partial q}\right)^{-1}\right) - \\ &\qquad \frac{\partial \overline{X}}{\partial q}\frac{1}{\overline{X}}\left(\frac{w\eta}{r+s} + (\alpha-1)^{\gamma}(\eta+r)\right) = 0 \end{aligned} \tag{41}$$

Proof. The Hamiltonian of the control problem is

$$\mathcal{H}(t) = e^{-rt}\left(y(t)^{1-\lambda} y_i(t)^{\lambda} - wE_i(t) - (\alpha-1)^{\gamma} j_i(t)\right) + \psi_1(t)\left(\eta(q,\theta_c) J_i(t) - sE_i(t)\right) + \psi_2(t)\left(\overline{X}(q,\theta_c,\alpha) J_i(t) - sy_i(t)\right) + \psi_3(t)\left(j_i(t) - \eta(q,\theta_c) J_i(t)\right).$$

The evolution of the co-state variables $\psi_1,\ \psi_2$ and ψ_3 satisfy respectively:

$$-e^{-rt}w - \psi_1 s = -\dot{\psi}_1, \tag{42}$$

$$e^{-rt}\lambda y^{1-\lambda} y_i^{\lambda-1} - \psi_2 s = -\dot{\psi}_2, \tag{43}$$

$$\psi_1\eta + \psi_2\overline{X} - \psi_3\eta = -\dot{\psi}_3\ . \tag{44}$$

The first order condition with respect to j_i is

$$-e^{-rt}(\alpha-1)^{\gamma} + \psi_3 = 0, \tag{45}$$

In the steady state, α (and so as q and θ_c) stays constant. Take time derivative of the above

$$re^{-rt}(\alpha-1)^{\gamma} + \dot{\psi}_3 = 0. \tag{46}$$

Then use (42) – (45) to eliminate the co-state variables:

$$\lambda y^{1-\lambda} y_i^{\lambda-1}\overline{X} - w\eta - (\alpha-1)^{\gamma}(\eta+r)(r+s) = 0 \tag{47}$$

which implies (38) in the text.

Next we derive the first order condition of the Hamiltonian with respect to α

$$-e^{-rt}\gamma(\alpha-1)^{\gamma-1} j_i + \psi_2 J_i \frac{\partial\overline{X}}{\partial\alpha} = 0. \tag{48}$$

Assuming the steady state (so that all state and control variables stay constant over time), take time derivative of the above and then use (42) – (46) and (50) to eliminate the co-state variables and j_i.

$$(r+s)\eta\gamma(\alpha-1)^{\gamma-1} - \lambda y^{1-\lambda} y_i^{\lambda-1}\frac{\partial\overline{X}}{\partial\alpha} = 0.$$

Finally use (47) to substitute out $\lambda y^{1-\lambda} y_i^{\lambda-1}$ yields (39).

The first order conditions of the Hamiltonian with respect to θ_c and q are respectively

$$-e^{-rt}E_i\frac{\partial w}{\partial \theta_c}+\psi_1 J_i\frac{\partial \eta}{\partial \theta_c}+\psi_2 J_i\frac{\partial \overline{X}}{\partial \theta_c}-\psi_3 J_i\frac{\partial \eta}{\partial \theta_c}=0,$$
$$-e^{-rt}E_i\frac{\partial w}{\partial q}+\psi_1 J_i\frac{\partial \eta}{\partial q}+\psi_2 J_i\frac{\partial \overline{X}}{\partial q}-\psi_3 J_i\frac{\partial \eta}{\partial q}=0.$$

Following the same procedure to derive (39) from (48) and then using (33) to evaluate $\partial w/\partial \theta_c$ and $\partial w/\partial q$ turn the above into (40) and (41) . ■

We have from (33) and (39) – (41) four equations in w, q, α and θ_c as functions of u^* which is pinned down by the free entry condition that each firm may only earn zero discounted profit in equilibrium. To derive this condition, first notice that from (34) – (36), we have in the steady state

$$\eta J_i = sE_i, \tag{49}$$
$$j_i = \eta J_i, \tag{50}$$
$$\overline{X} J_i = s y_i. \tag{51}$$

Hence the firm's expense at each t may be expressed as

$$wE_i(t)-(\alpha-1)^{\gamma} j_i(t)=\frac{y_i}{\overline{X}}\left(w\eta+(\alpha-1)^{\gamma} s\eta\right)$$

And using (38), the cash flow from (37) is given by

$$y\left(\frac{\lambda\overline{X}}{w\eta+(\alpha-1)^{\gamma}(\eta+r)(r+s)}\right)^{\frac{\lambda}{1-\lambda}}\left(1-\lambda\frac{w\eta+(\alpha-1)^{\gamma}\eta s}{w\eta+(\alpha-1)^{\gamma}(\eta+r)(r+s)}\right)=rc_e \tag{52}$$

which is equal to the interest expense of entry in equilibrium.

To complete the solution of the equilibrium, I now turn to the determination of aggregate output y. In the steady state, job creation is equal to job destruction at each point in time. Let u be the rate of unemployment

$$JC = uH\frac{\eta}{q} = s(1-u)H = JD.$$

Solving for u

$$u = \frac{sq}{\eta + sq}, \tag{53}$$

and this implies an aggregate employment of

$$\widetilde{E} = H \frac{\eta}{\eta + sq}.$$

With output of intermediate good per employed worker still given by $\overline{X}/\eta$, aggregate output is equal to

$$y = N^{\frac{1}{\lambda}} \frac{H}{N} \frac{\overline{X}}{\eta + sq}. \tag{54}$$

By definition, the job queue is

$$q = \frac{uH}{NJ_i}.$$

Using (53) for u and (38) and (51) for J_i, the above becomes

$$y = \frac{H}{N} \frac{\overline{X}}{\eta + sq} \left(\frac{w\eta + (\alpha - 1)^{\gamma} (r + s)(\eta + r)}{\lambda \overline{X}} \right)^{\frac{1}{1-\lambda}}. \tag{55}$$

Equations (54) and (55) imply

$$N = \left(\frac{w\eta + (\alpha - 1)^{\gamma} (r + s)(\eta + r)}{\lambda \overline{X}} \right)^{\frac{\lambda}{1-\lambda}}$$

from which it follows from (54) that

$$y = H \frac{w\eta + (\alpha - 1)^{\gamma} (r + s)(\eta + r)}{\lambda (\eta + sq)}. \tag{56}$$

We now have the complete characterization of the equilibrium given by (33), (39) – (52) and (56).

The system is too complicated to be subject to a qualitative analysis. And we may only study the properties of the equilibrium through numerical experiments. As benchmark parameter values for the following, I set $r = s = 0.05$, $c_e = 1$, $\gamma = 1.5$ and $\lambda = 0.8$ where the values for γ and λ are chosen to satisfy assumption (A3). The comparative steady states are summarized in the various panels of fig.3.

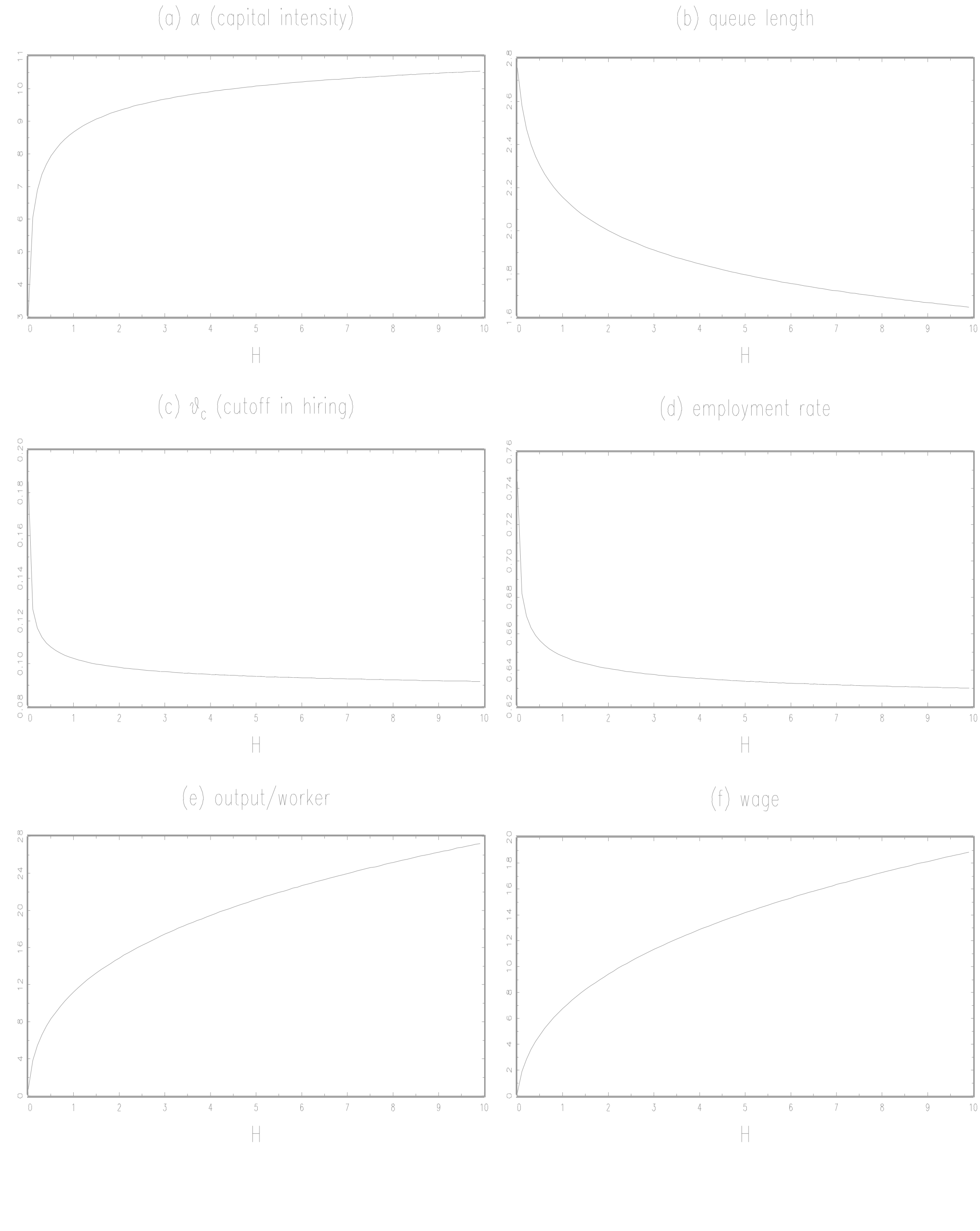

Fig. 4. Comparative steady states

First, capital intensity is increasing in market size, just as in the two–period economy. However, the second panel shows that the job queue is now falling with market size. Evidently, assumption (A3) is not sufficient to guarantee that the positive effect on the job queue of more capital intensive jobs being created dominates the negative effect on the job queue of job creation becoming more profitable. However, this does not imply that there would be greater employment in the steady state. With more capital intensive and specialized jobs created, firms are obliged to be more selective in accepting job applications as evidenced by the downward sloping relationship between θ_c and market size in panel (c). In the present case, the steady state employment indeed falls with market size as the effect of more exact match requirement on employment dominates the opposing effect of the availability of more job vacancies (shorter queue length). However, the falling employment is accompanied by greater output per worker and a higher wage rate as the last two panels testify. Not shown is that the utility of an unemployed worker as well as the product variety are also increasing in market size. In all, the effects of market size on labor market outcomes and productivity are essentially identical to the effects in two-period economy, with the only difference being how the length of the job queue varies with market size.

In the infinite horizon economy, the steady state equilibrium depends, among other exogenous variables, on the interest rate and the rate of job destruction–two considerations that do not apply previously in the two–period economy. It is thus not surprising that the restrictions on γ and λ imposed by assumption (A3) do not guarantee that the job queue would lengthen with market size even though they do in the two–period economy. It is only when r and s are also chosen appropriately that we may establish the same conclusion. Intuitively, the result would obtain when the productivity gains from greater product variety spur a major increase in the capital intensity of the jobs offered. For then the effect of the rising match requirement would dominate the market tightness effect. A smaller interest rate turns the investment decision more elastic to the increase in the marginal revenue under which the capital intensity effect should be stronger and be more likely to dominate.

The effect of s is harder to evaluate as it should affect investment incentives and market tightness in the same way. On the one hand, a larger s should make the investment decision less elastic to changes in the marginal revenue for the payoff to create capital intensive jobs should diminish when these jobs are less likely to last. But the same force should apply equally to the market tightness effect since any jobs created, capital intensive or otherwise, are less likely to last in any case.

My numerical experiments suggest that it is at a larger s that the capital intensity effect dominates. Holding s at the benchmark of 0.05, I find that the job queue is still decreasing in market size for an interest rate as small as $r = 0.01$. However, when s is allowed to increase to 0.15, the relationship between the job queue and market size is turned around for an interest rate that may be as high as $r = 0.03$. The comparative steady states are reported in the various panels of fig. 4 for this combination of $\{r, s\} = \{0.03, 0.15\}$. The qualitative results are essentially identical to the benchmark, except that in panel (b), the queue length is now increasing in market size, just as in the two–period economy when assumption (A3) is satisfied. More interestingly, when compared to the results in fig.3, we can see that at each H, the job queue is longer. In other words, fewer jobs (per unemployed) are offered. The lowering of the interest rate quite unexpectedly does not help raise employment. The beneficial effect is solely reflected in the increase in the capital intensity of the jobs created. But wages are higher as a result as evidenced by a comparison between the last panels in fig.3 and fig.4. In fact, we can see from comparing panels (e) of fig.3 and fig.4 that the increase in the capital intensity of the jobs created suffices to raise output per worker even though employment has fallen.

The effects of the interest rate on how employment may respond to market size as well as on employment itself should work in the opposite direction too. At a

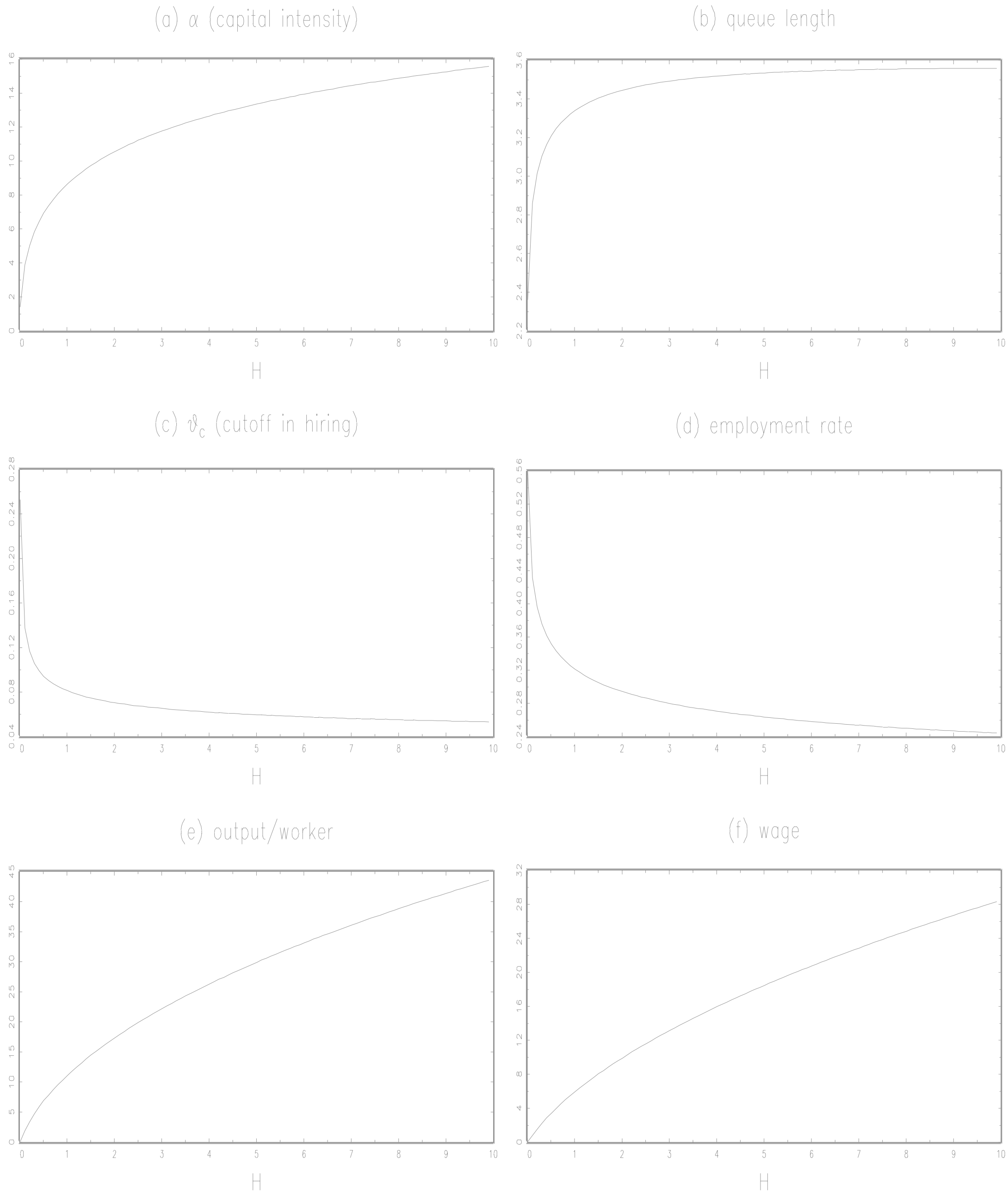

Fig. 5. Comparative steady states with increasing job queue

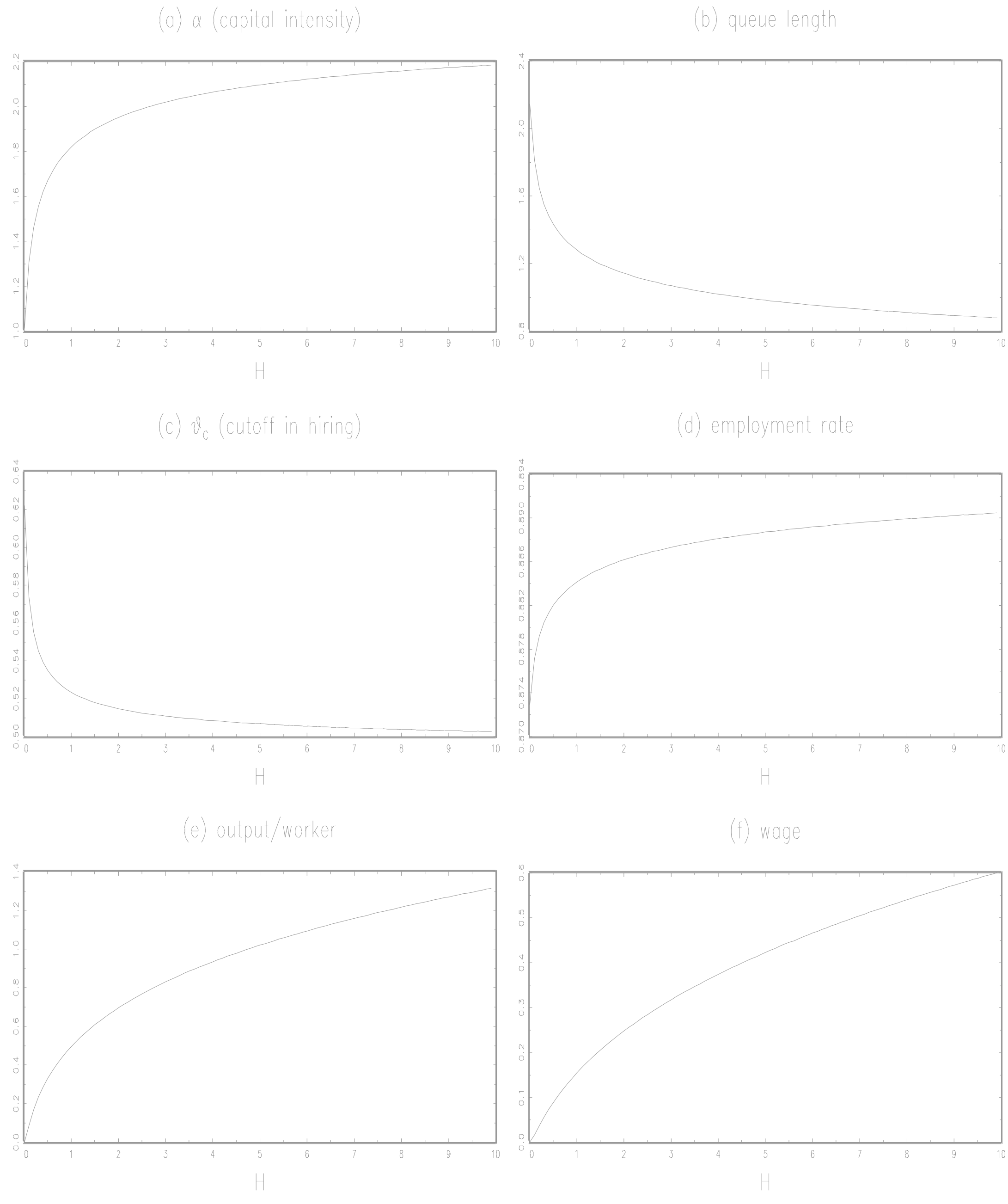

FIG. 6. Comparative steady states: increasing employment

higher interest rate, the investment decision becomes less elastic to changes in the marginal revenue. Not only that the job queue may shorten as market size increases just as in the benchmark, but also employment may rise if the market tightness effect dominates the now weakened capital intensity effect. To verify the intuition, I repeat the comparative steady states with an interest rate $r = 0.2$. The results are reported in the various panels of fig.5. At this high interest rate, the job queue is falling with market size, a return to the relationship in the benchmark. Furthermore, the greater job creation now in fact results in greater employment. The rising employment however does not make workers better off. At each H, wages and output per worker are lower when compared to the benchmark shown in fig.3. The effects of the interest rate on employment and output are similar to the effects of market size. Once again, output and employment may deviate in opposite directions as a low interest rate may raise output but not necessarily employment.