Market Size, Productivity and Employment with Labor Market Frictions

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Abstract

That workers in cities are more productive is arguably the most celebrated fact in urban economics. The available evidence, however, suggests that agglomeration economies do not appear to extend to helping raise employment in cities above less dense locales. But in the absence of other effects, standard economic reasoning suggests that more productive workers should also experience higher employment. This paper explains that the outward shift in the production function by improving investment incentives would also induce firms to create more specialized jobs. In equilibrium, the productivity gains may not be dissipated in more job creation, but in the creation of more specialized jobs that may only be sustained in a labor market with a longer job queue.

Keywords: job matching, agglomeration economies, unemployment.

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1. INTRODUCTION

That the agglomeration of economic activities helps raise productivity is a major theme in urban economics for which there is ample empirical support.[1] The available evidence, however, suggests that its effect on employment, positive or otherwise, is far from clear. The study by Alperovich (1993) did find that larger cities tend to have lower unemployment in a sample of Isaeli cities. But in two early studies, Vipond (1974) and Sirmans (1977) uncover positive relationships between the rate of unemployment and city size in a sample of British cities. More recently, Maillard (1997) finds an inverted–U relationship between unemployment and city size among French cities, where unemployment is increasing in city size up to cities with a population of one million. Furthermore, Glaeser et al. (1995) find that the raw correlation between unemployment and population is essentially zero in a sample of 203 US cities. Though they did not report the conditional correlations, their discussion

> "...Southern cities in the sample tend to have relatively smaller populations, ...dramatically lower per capita incomes, lower unemployment... Northeastern cities are large, have...higher per capita income, higher unemployment..."

suggests that the conditional correlations between unemployment and city size may well be positive.

Standard economic reasoning, however, suggests that if density has made workers more productive for whatever reason, in the absence of other effects, it should raise employment as well. The point is most easily seen in a model of a frictionless labor market with an upward sloping labor supply curve, where the increase in productivity shifts out the labor demand curve, raising wages and employment. The same conclusion applies equally well in a model of a frictional labor market, with the matching of

[1] For more recent evidence, see Rauch (1993) and Ciccone and Hall (1996).

workers and job vacancies governed by some constant returns to scale job matching function. To see this, let f be the production function, v and u the numbers of job vacancy and unemployed workers respectively and $m(v,u)$ the job matching function. In equilibrium, free entry equates the expected payoff to job creation to the cost of posting a job vacancy:

$$m(1,q)(f-w)=rc.$$

where $q=u/v$ and w, r and c have the usual meanings. With labor market frictions, any productivity increase that results in an outward shift of the production function f will not in general be entirely dissipated in higher wages, but will rather be shared between the firm and the worker in a job match. The remaining channel for the higher payoff to job creation to be competed away is via more job creation, lowering the probability that each vacancy will be filled. Common to both models is that the productivity gains are partially dissipated in higher wages and partially in increased employment. Hence for employment not to increase, either the greater job creation will not result in higher employment or the higher payoff to job creation is dissipated away in some other channels or both. What might be the reasons?

In a small town, jobs are usually less capital intensive as well as less specialized. An average worker would qualify quite well for many different types of jobs.[2] On the other hand, jobs in cities are usually more capital intensive and more specialized. A typical job vacancy may only find a good match in workers that possess some specific skills. In this way, there may not be greater job creation or the increased job creation may not result in higher employment if the jobs are more specialized and so harder to fill. These are certainly not novel ideas. The task is to construct

[2]See Hall (1989). In the movie Local Hero, the innkeeper in the small Scottish seaside town where the oil company executive Burt Lancaster plans to build an oil refinery doubles as the mayor as well as the town's only certified accountant. While it is difficult to verify how representative this folklore is, it is probably fair to say that it is far from being a figment of the scriptwriter's imagination.

an equilibrium theory of the labor market in which firms would be inclined to create more specialized jobs in a denser labor market, and more importantly to establish that this would indeed give rise to the possibility that employment may fall in the midst of rising productivity.

The labor market model I study in this paper is based on the wage–posting model first proposed by Peters (1991) and Montgomery (1991) in which search frictions arise because of the lack of coordination in workers' applications to job vacancies that may result in some vacancies receiving none but some others more than one applicants. To it, I add heterogeneous job matches with which the output of a job match depends continuously on the *quality* of the match. The main element in the model is the tradeoff the firm faces in choosing between more capital intensive and specialized jobs and less capital intensive ones. In the absence of match imperfections, more capital intensive jobs are more productive. But these jobs are also more exact in their skill requirements and less tolerant of any match imperfections. Indeed, these jobs may yield very little output if not matched to workers with the appropriate skills.

The outward shift of the production function due to the productivity gains from increasing density would raise investment incentives and eventually aggregate output. Employment may decline in the mean time, however, because the more capital intensive and specialized jobs created are harder to fill. In addition, there may in fact be fewer jobs created in equilibrium. This happens because with search frictions, the more capital intensive jobs created are only viable when each attracts a greater number of applicants on average. With few applicants to choose from, firms are unlikely to find any acceptable matches for the now more specialized vacancies to make good use of the more capital intensive technology. In this way, the productivity gains may not be dissipated in more job creation and higher employment but in the creation of jobs that are harder to fill. Nevertheless, aggregate output will increase because of the productivity gains from greater density, as well as the creation of more capital intensive jobs.

While the channel through which agglomeration economies operate is still an area of active research, two leading hypotheses have emerged. The first one, first proposed by Jane Jacobs (1968) and made well known to economists in Lucas (1988), is the human capital externality that arises through the day to day interactions among workers locating in close proximately. The second one is the returns to market size in production with fixed costs, which has been the workhorse in recent research in economic geography.[3] In this paper, I choose to motivate the relationship between market size and productivity via this channel. The model is a richer model, with the productivity of workers, product variety and the job queue all endogenously determined and interdependent.

The relationship between productivity and employment in a frictional labor market has been previously studied in Mortensen and Pissarides (1998). They focus on how the *growth rate* of productivity affects employment, while eliminating any possible relationship between the level of productivity and employment by assuming that the cost of job creation is straightly proportional to the former. Caballero and Hammour (1998) also study a model whereby the output growth is accompanied by falling employment. However, the output increase is due to exogenous technological change and the falling employment is merely the equilibrium response to workers acquiring greater bargaining power. The paper is closest to Acemoglu and Shimer (1999b) who also entertain the notion that more capital intensive jobs are more specialized jobs that are harder to fill in their study on how the provision of unemployment insurance may raise output.

The next two sections present the two–period version of the model and establish the possibility that market size may raise productivity but lower employment. The empirical evidence does not point to a negative relationship between market size and

[3]See Masahisa, Krugman and Venables (1999) for a most updated survey.

unemployment, but rather the lack of any systematic relationship. Increases in productivity, by virtual of raising the payoff to job creation, tend to raise employment in the absence of other effects. The model explains how one such effect works to counter this tendency. By showing how a negative relationship between unemployment on the one hand and market size and productivity on the other hand may emerge, the analysis establishes that this effect suffices to subdue the positive effect on employment of job creation being more lucrative. Section 4 generalizes the analysis to an infinite horizon economy. Besides demonstrating the results obtained for the two–period model hold in an infinite horizon economy, this section analyzes the role the interest rate plays in how market size would affect employment. Section 5 concludes.

2. MODEL

The model is a closed economy populated by a continuum of workers of measure H. There are two periods in the model. In the first period, firms enter, invest in physical capital, post jobs and announce wages. In the second period, workers apply to jobs after which job matches are formed. Then production commences, wages are paid and the period is over.

A. Monopolistic competition

There is a total of N (to be determined endogonously) monopolistically competitive firms in the city, each of which produces a differentiated intermediate good. Let y_i be the quantity of the ith intermediate good. The production function for the final good is

$$y = \left(\int_0^N y_i^{\lambda} di \right)^{\frac{1}{\lambda}}. \tag{1}$$

With the final good sector perfectly competitive and the price normalized to one, each intermediate good firm faces the (inverse) demand curve:

$$p_i = y^{1-\lambda} y_i^{\lambda-1}.$$

There is a fixed cost equal to c_e that each of the N firms will have to incur at the time of entry.

B. Technology

Each monopolistically competitive firm $i \in [0, N]$ may post some J_i jobs, each of which may be matched with one and only one worker. If k is the capital installed for the job, the output of a worker–job match is given by

$$X = k(1 - \alpha\theta).$$

with the cost of capital investment equal to $c_k(k) = k^\gamma$ for some $\gamma > 1$. Each firm uses a product–specific technology that would be most productive when matched with workers who possess some specific skills. The productivity of a worker–firm match therefore is idiosyncratic. The variable θ is to measure the quality of a worker–firm match with $\theta = 0$ a perfect match. For simplicity, I assume that θ is uniformly distributed on the unit interval and is iid across any possible worker–firm pairs. The exact value of θ for a given match is not known until the firm and worker make contact so that it is not possible for firms and workers to direct their searches toward the best possible match.[4]

The parameter α gauges the percentage output loss due to the worker-firm mismatch. More capital intensive jobs are usually jobs that use more advanced technologies and these jobs are typically more exact in their skill requirements. Hence,

[4]In princple, the results to follow should still hold if searches are directed but as long as some imperfections remain in the knowledge of θ before the pair makes contact.

the output loss due to the worker–firm mismatch would be greater for more capital intensive jobs, from which it follows that α should be a positive function of k. For analytical convenience I shall work with the following parameterization:

(A1) $\alpha = k + 1.$

This assumption implies that firms always choose an $\alpha > 1$ that serves to simplify the calculations to follow. I shall further assume that the returns to variety parameter in the production of the final good

(A2) $\lambda \in \left(\frac{1}{2}, 1\right).$

The assumption is a more stringent restriction on the usual $(0, 1)$ range that λ may take in models with monopolistic competition. This further restriction is necessary for the social optimum to be well defined.[5]

C. Search frictions

Workers make their job applications after firms make capital investment, post vacancies and announce wages. The matching of workers and job vacancies follow the wage–posting model of Montgomery (1991) and Peters (1991) where each worker may apply to no more than a single job opening. In the absence of coordination among workers in their job applications, a job opening may thus have none, one or more than one applicants. Conversely a given worker may not land a job if he happens to apply to a vacancy that attracts at least one other applicants.

The present analysis differs from the existing models for firms may find it optimum not to hire for a given vacancy at all if none of the applicants are deemed to be an acceptable match. This happens when the marginal revenue yielded by even the best

[5] In the absence of search frictions and without capital investment, aggregate output net of the costs of entry is equal to $y = HN^{\frac{1}{\lambda}-1}\overline{x} - c_e N$ where $\overline{x}$ is average output/worker. Optimum N exists only when $\frac{1}{\lambda} - 1 < 1$ or $\lambda > \frac{1}{2}$. It will be seen in the following that the same restriction applies in the presence of search frictions and capital investment.

match falls below the posted wage. In particular, the firm may set a minimum match requirement θ_c and not hire at all for the vacancy if not a single applicant has a match parameter $\theta \leq \theta_c$.

Suppose a given vacancy has received n applications. The best match among the n applicants would then be some $\theta_n = \min_{j=1,\ldots,n} \{\theta_j\}$ that has a CDF given by $G_n(\theta_n) = 1 - (1-\theta_n)^n$ since each θ_j is uniformly distributed on the unit interval. The probability that the vacancy would successfully recruit is therefore equal to $1 - (1-\theta_c)^n$ and the expected output that the vacancy yields is given by

$$X_n = (1 - (1-\theta_c)^n)\, k\, (1 - \alpha E[\theta_n \mid \theta_n \leq \theta_c]) \tag{2}$$

where

$$E[\theta_n \mid \theta_n \leq \theta_c] = \frac{(1+n)^{-1}\left(1 - (1-\theta_c)^{1+n}\right) - (1-\theta_c)^n \theta_c}{1 - (1-\theta_c)^n} \tag{3}$$

is the conditional expectation of θ_n given that $\theta_n \leq \theta_c$. Suppose each of some h workers apply to the given vacancy with probability p, the expected output the vacancy may eventually generate is therefore

$$\overline{X} = \sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} X_n \tag{4}$$

and the probability that the vacancy will successfully recruit is

$$\eta = \sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} (1 - (1-\theta_c)^n).$$

If each of the h workers apply to each of some J vacancies with equal probability, then $p = \frac{1}{J}$. Let $q = \frac{h}{J}$ be the average queue length the vacancy will attract and when h becomes large it can be shown that

Lemma 1

(a) $\overline{X}(q, \theta_c, k, \alpha) = k\left(1 - e^{-q\theta_c}\left(1 - \alpha\theta_c - \frac{\alpha}{q}\right) - \frac{\alpha}{q}\right)$,

(b) $\eta(q, \theta_c) = 1 - e^{-q\theta_c}$.

Proof. In appendix A.

I have in the above assumed that each vacancy posted by a given firm recruits separately. This treatment entails some loss of generality. For example suppose the firm posts two vacancies and one attracts two good matches and the other none. Then the above calculation assumes that only one vacancy would be filled. However if the firm can coordinate the recruitments among its vacancies, then both vacancies should be filled in this case.[6] The matching technology of the wage–posting model thus exhibits a certain increasing returns to scale at the firm level. I choose not to deal with this complication for the model quickly becomes intractable in doing so.

D. Profit maximization and equilibrium in wage posting

Although each vacancy that the firm posts may only successfully recruit with a probability less than one and the output yielded by each filled job is uncertain, in posting a continuum of job vacancies, these uncertainties vanish at the firm level. The firm will recruit for certain $J_i\eta\left(q,\theta_c\right)$ workers and produce $J_i\overline{X}\left(q,\theta_c,k,\alpha\right)$ units of output. For a given average queue length, a posted wage, a capital intensity and a minimum match requirement, the ith monopolistically competitive firm chooses the number of vacancies to post to maximize[7]

$$\pi_i\left(q,\theta_c,w,k,\alpha\right)=\max_{J_i}\left\{y^{1-\lambda}\left(J_i\overline{X}\left(q,\theta_c,k,\alpha\right)\right)^{\lambda}-J_i\left(w\eta\left(q,\theta_c\right)-k^{\gamma}\right)\right\}.$$

[6]This type of increasing returns is first noted by Arrow (1971) in his famous example where the expected output of a firm with 2 workers and 4 machines would be greater than that of a firm with 1 worker and 2 machines where each machine breaks down with some exogenous probability.

[7]If the firm knows for certain that it may only fill $J_i\eta$ jobs, then it seems that it should only install capital for that many jobs, but not for all posted vacancies. However, under the assumption that the individual vacancies recruit separately without coordination, it is only appropriate to assume the firm has to install capital for each posted vacancy.

This results in

$$J_i(q,\theta_c,w,\alpha) = y\left(\frac{\lambda\overline{X}(q,\theta_c,\alpha)^{\lambda}}{w\eta(q,\theta_c)}\right)^{\frac{1}{1-\lambda}}, \tag{5}$$

$$\pi_i(q,\theta_c,w,\alpha) = y\left(\frac{\lambda\overline{X}(q,\theta_c,\alpha)}{w\eta(q,\theta_c)+(\alpha+1)^{\gamma}}\right)^{\frac{\lambda}{1-\lambda}}(1-\lambda). \tag{6}$$

where I have used (A1) that $k=\alpha+1$ to substitute out k.

Let $\mu(q,\theta_c)$ equal to the probability that a worker will successfully obtain a job in applying to firms whose vacancies attract an average queue of q and which only accept matches with $\theta \leq \theta_c$. Assuming risk-neutral workers, the worker's expected utility is

$$u(q,\theta_c,w) = \mu(q,\theta_c)\,w. \tag{7}$$

Any triples of (q,θ_c,w) offered in equilibrium must yield workers the same expected utility for any triples that offer less attract no applicants. Let u^* be the level of utility workers obtain in equilibrium. Then any triples of (q,θ_c,w) offered in equilibrium must satisfy:

$$w = \frac{u^*}{\mu(q,\theta_c)}. \tag{8}$$

Next we can establish that

Lemma 2 $\mu(q,\theta_c) = \eta(q,\theta_c)/q$.

Proof. In appendix A.

By lemma 2, (6), (7) and (8), we may state the firm's profit maximization as in

$$\pi_i(u^*) = \max_{\{\alpha,q,\theta_c\}}\left\{y\left(\frac{\lambda\overline{X}(q,\theta_c,\alpha)}{u^*q+(\alpha+1)^{\gamma}}\right)^{\frac{\lambda}{1-\lambda}}(1-\lambda)\right\} \tag{9}$$

where the choice variables are technology α, queue length q and minimum match requirement θ_c. It may appear unusual that the queue length could be a choice variable for the firm. But by virtue of (8), this is merely equivalent to stating the firm's maximization in terms of choosing a wage and a match requirement, which implies a certain average queue length given u^*. The calculations to follow are simpler when the maximization is stated in terms of q however. It follows immediately that optimal

$$\theta_c^* = \max_{\theta_c} \left\{ \overline{X}(q, \theta_c, \alpha) \right\}$$

and from lemma 1a, we have $\theta_c^* = \alpha^{-1}$ which means that in equilibrium, the firm would hire as long as the best match for a vacancy yields positive output. Substitute θ_c^* back to lemma 1a, $\overline{X}$ is simplified to

$$\overline{x}(q, \alpha) = (\alpha - 1) \left(1 - \frac{\alpha}{q} \left(1 - e^{-q/\alpha} \right) \right). \tag{10}$$

and η becomes

$$\eta \left(\frac{q}{\alpha} \right) = 1 - e^{-q/\alpha}. \tag{11}$$

In addition to $\theta_c^* = \alpha^{-1}$, we now have from the maximization in (9), as functions of u^*, the firm's optimal α and q which together imply a wage offer from (8). The final step is to solve for u^* via a free entry condition that firms in the intermediate good sector must earn zero profit in equilibrium:

$$\pi_i(u^*) = c_e.$$

Evidently, this procedure is equivalent to solving[8]

$$u^* = \max_{\{q, \alpha, w\}} \left\{ \frac{\eta\left(\frac{q}{\alpha}\right) w}{q} \right\} \tag{12}$$

subject to the firms earning zero profit:

$$y \left(\frac{\lambda \overline{x}(q, \alpha)}{w\eta\left(\frac{q}{\alpha}\right) + (\alpha + 1)^\gamma} \right)^{\frac{\lambda}{1-\lambda}} (1 - \lambda) = c_e. \tag{13}$$

Solving for ηw in the above and substitute the result back into (12), the problem simplifies to

$$u^* = \max_{\{q, \alpha\}} \left\{ \lambda \left(\frac{(1-\lambda) y}{c^e} \right)^{\frac{1-\lambda}{\lambda}} \frac{\overline{x}(q, \alpha)}{q} - \frac{(\alpha - 1)^\gamma}{q} \right\} \tag{14}$$

[8]This equivalence is discussed in details in Acemoglu and Shimer (1999a,b) and Moen (1997).

In the above, the two terms $\overline{x}/q$ and $(\alpha-1)^{\gamma}/q$ denote respectively the expected output of intermediate good per worker and capital investment per worker. That the first term is augmented by the factor $\lambda\left(\frac{(1-\lambda)y}{c^e}\right)^{\frac{1-\lambda}{\lambda}}$ reflects the fact that the marginal revenue of a unit of intermediate good to the monopolistically competitive firms depends on aggregate output, the fixed cost of entry as well as the price elasticity of demand these firms face. The first order conditions for q and α are respectively

$$\lambda\left(\frac{(1-\lambda)\,y}{c^e}\right)^{\frac{1-\lambda}{\lambda}}\frac{\partial\left[\overline{x}\,(q,\alpha)\,/q\right]}{\partial q}-\frac{\partial\left[(\alpha-1)^{\gamma}/q\right]}{\partial q}=0, \tag{15}$$

$$\lambda\left(\frac{(1-\lambda)\,y}{c^e}\right)^{\frac{1-\lambda}{\lambda}}\frac{\partial\left[\overline{x}\,(q,\alpha)\,/q\right]}{\partial \alpha}-\frac{\partial\left[(\alpha-1)^{\gamma}/q\right]}{\partial \alpha}=0. \tag{16}$$

The solution of the equilibrium is yet to be completed since aggregate output y is yet to be specified. To do so, I now turn to the general equilibrium.

E. The general equilibrium

Let $\widehat{x}$ be the average output of intermediate good yielded by a filled vacancy, then by definition $\overline{x}=\widehat{x}\eta$ from which it follows

$$\widehat{x}=\frac{\overline{x}}{\eta}.$$

And this must clearly equal to the average output of intermediate good an employed worker produces. With each worker employed with probability $\mu=\eta/q$, total employment is equal to

$$E=H\frac{\eta}{q}.$$

From (1), aggregate output may be expressed as

$$y=N^{\frac{1}{\lambda}}\frac{H}{N}\frac{\overline{x}}{q} \tag{17}$$

In equilibrium, the job queue must satisfy $q=H/NJ_i$ and using (5) it becomes

$$q=\frac{H}{Ny}\left(\frac{\lambda\overline{x}^{\lambda}}{w\eta+(\alpha+1)^{\gamma}}\right)^{\frac{1}{\lambda-1}}. \tag{18}$$

Equations (17), (18) and the zero profit condition (13) imply

$$y = \frac{N}{1-\lambda} c_e, \tag{19}$$

$$N = \left(\frac{H(1-\lambda)}{c_e} \frac{\overline{x}}{q} \right)^{\frac{\lambda}{2\lambda - 1}}. \tag{20}$$

The first equation states that aggregate output is increasing in product variety while the second one states that for a given output of intermediate good per worker, product variety rises with market size H normalized by the cost of entry c_e. The latter relationship captures the well–known increasing returns to scale associated with models with monopolistic competition and fixed costs of entry. A larger market allows the fixed cost to be spread among greater units of output and this makes product variety increasing in market size in equilibrium. The equilibrium is now completely specified by (19), (20), (15) and (16).

G. Comparative statics

I should now move on to checking how increases in market size H affect productivity, technology choice and employment. First evaluating the partial derivatives in (15) and (16) and then combining the two equations yield

$$q = (\gamma - 1)^{-1} \frac{\widehat{q} \left(1 - e^{-\widehat{q}} - \widehat{q} e^{-\widehat{q}}\right)}{\widehat{q} - 2\left(1 - e^{-\widehat{q}}\right) + \widehat{q} e^{-\widehat{q}}} \tag{21}$$

where $\widehat{q} = q/\alpha$. As it embeds the first order condition for α, I will call the above the technology choice condition. A few properties of this equation will be useful.

Lemma 3 *In* (21),

(i) q *is decreasing in* $\widehat{q}$ *for* $\widehat{q} > 0$,

(ii) $\lim_{\widehat{q} \to 0} q = 3(\gamma - 1)^{-1}$,

(iii) $\lim_{\widehat{q} \to \infty} q = (\gamma - 1)^{-1}$.

Proof. In appendix A.

Notice that a negative relationship between q and $\widehat{q}$ implies a positive relationship between q and α. Intuitively, the positive relationship is explained by the fact that a longer job queue is necessary to sustain the optimality of more capital intensive and specialized jobs as these jobs are less forgiving in match imperfections.

Next use the definition $\widehat{q} = q/\alpha$ and apply (19) and (20) to (15)

$$\lambda \left(\frac{H(1-\lambda)}{c_e} \left(1 - \frac{1-e^{-\widehat{q}}}{\widehat{q}} \right) \right)^{\frac{1-\lambda}{2\lambda-1}} \left(2\frac{1-e^{-\widehat{q}}}{\widehat{q}} - e^{-\widehat{q}} - 1 \right) + \left(\frac{1}{\widehat{q}} - \frac{1}{q} \right)^{\gamma - \frac{\lambda}{2\lambda-1}} q^{\gamma-1} = 0. \tag{22}$$

This is the condition that pins down the job queue that yields firms zero profit while maximizes workers' utility for each α and will be referred to as the market tightness condition in the following. Under the following assumption

(A3) $\gamma - \frac{\lambda}{2\lambda-1} \geq 0$,

Lemma 4

(i) Equation (22) *implies a well-defined positive valued function* $q = F(\widehat{q}) > \widehat{q}$ *for* $\widehat{q} > 0$,

(ii) $dF(\widehat{q})/d\widehat{q} > 0$,

(iii) $\lim_{\widehat{q}\to 0} F(\widehat{q}) = 0$,

(iv) $F(\widehat{q})$ *unbounded.*

Proof. In appendix A.

A positive relationship between $\widehat{q}$ and q implies that an increase in α would be followed by a decrease in q.[9] We may interpret the negative relationship as reflecting the fact that when the technology becomes more productive, the market will respond by offering more vacancies, shortening the job queue as a result.

[9]Note that a decrease in q may not be followed by an increase in α from the given positive relationship between $\widehat{q}$ and q and so the analysis cannot be readily translated into the $q \times \alpha$ space.

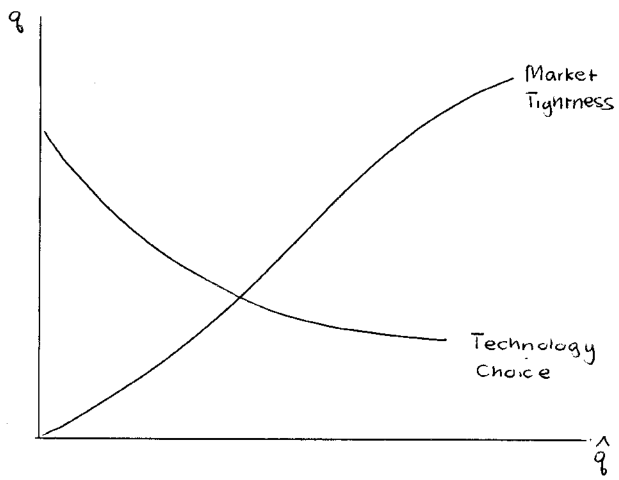

FIG. 1. The determination of equilibrium

Figure 1 depicts q as functions of $\widehat{q}$ from the technology choice and market tightness conditions. Clearly a unique equilibrium exists.

As to the comparative statics with respect to market size H, first notice that the technology choice condition is independent of H. As to the market tightness condition, the left side of (22) is decreasing in H and $\widehat{q}$. Thus $\widehat{q}$ must fall to restore the equality in the equation at a larger H. The market tightness condition shifts in to the left as shown in fig.2. At a given technology, the decline in $\widehat{q}$ is merely the fall in q in response to job creation becoming more profitable due to the increase in market size. In the absence of the technology choice, more jobs will be created, raising employment as a result. But as fig.2 testifies, with the downward sloping technology choice condition, the conclusion is turned around as the job queue will instead lengthen. With $\widehat{q}$ fallen but q becoming longer, α must have increased in the mean time. And indeed this explains why the productivity gains are not dissipated in more job creation since the jobs created in the new environment need a longer job queue to be viable. This

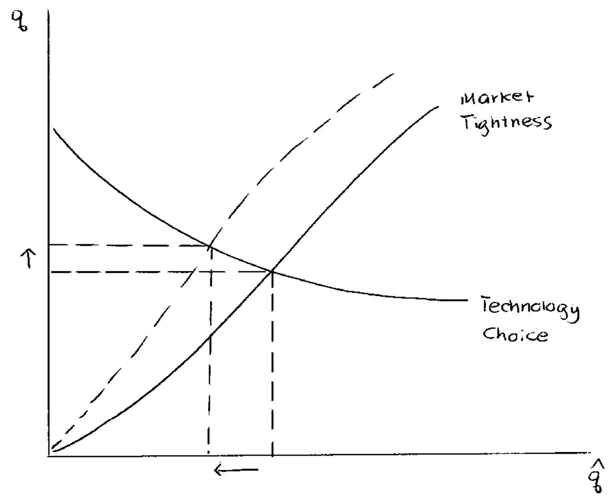

FIG. 2. Comparative statics of market size

will cause an unambiguous decline in equilibrium employment since not only fewer jobs are created but also the jobs created are more exact in match requirement. We may summarize the results in the following proposition.

Proposition 1 *In equilibrium, both the capital intensity of the jobs created and the job queue are increasing in market size H. In the mean time, employment declines.*

The decline in employment, however, is not so much a perverse outcome because

Proposition 2 *Product variety, average output per worker, the expected output of intermediate good yielded by a vacancy, wages and workers' expected utility all rise as market size H increases.*

Proof. In appendix A.

The normal channel whereby increases in market size raise product variety and then the marginal revenue to the monopolistically competitive firms and eventually aggregate output is still operative. It is now just that when the firms choose to invest in more capital intensive jobs, they choose to offer fewer of them. Nevertheless, the effect of the increase in capital intensity on output suffices to dominate the effect of the decline in employment.

The role of the technology choice in driving the negative relationship between market size and employment deserves further attention. For a given q, the more capital intensive jobs created would only recruit with a lower probability and there would also be greater penalty for a given mismatch. A longer job queue serves to compensate firms for the falling hiring probability and helps lower the expected mismatch. Given the parameter restriction in assumption (A3), the effect of the technology choice on the job queue will dominate the effect of the productivity gains due to greater product variety. The increase in marginal revenue is dissipated not in terms of more jobs offered but in terms of the jobs becoming less likely to find good matches for a given queue length. In particular, even though the job queue has lengthened, the hiring probability $\eta = 1 - e^{-\widehat{q}}$ which only depends on $\widehat{q}$ instead of increasing would go down. With or without the technology choice, the jobs may only be filled with lower probabilities in equilibrium which is only to be expected when job creation is now a more profitable business.

Finally, it is useful to note that it is not an inevitable outcome of the model that employment will fall with market size. Without assumption (A3), the market tightness condition will become downward sloping over a certain range of $\widehat{q}$. In this case, the increase in market size may lower the job queue and raise employment as in a model without the technology choice. In all, the analysis serves to demonstrate the possibility that agglomeration economies do not necessarily result in higher employment.

3. SOCIAL OPTIMUM

The papers by Moen (1997) and Acemoglu and Shimer (1999a) have shown that in the absence of other distortions, both the number of jobs offered in equilibrium and firms' capital investment in the wage–posting model satisfy constrained optimality. With monopolistic competition, the equilibrium necessarily deviates from constrained optimality. The more interesting question is in what way the distortions introduced by market power and the inefficient entry of firms in monopolistic competition affect labor market outcomes.

Given the search frictions, aggregate output net of capital investment and the cost of entry is

$$S = \max_{\alpha, q, \theta_c} \left\{ N^{\frac{1-\lambda}{\lambda}} H \frac{\overline{X}\left(q, \theta_c, \alpha\right)}{q} - H \frac{\left(\alpha - 1\right)^{\gamma}}{q} - c_e N \right\}. \tag{23}$$

As in equilibrium, the optimal minimum match requirement satisfies

$$\theta_c^o = \max_{\theta_c} \left\{ \overline{X}\left(q, \theta_c, \alpha\right) \right\}$$

and this allows substituting $\overline{X}\left(q, \theta_c, \alpha\right)$ in the above by $\overline{x}\left(q, \alpha\right)$ given in (10). The first order condition with respect to N yields[10]

$$N = \left(\frac{\left(1 - \lambda\right) H}{c_e \lambda} \frac{\overline{x}\left(q, \alpha\right)}{q} \right)^{\frac{\lambda}{2\lambda - 1}}. \tag{24}$$

For given $\overline{x}/q$, the socially optimum N exceeds its value in equilibrium as given in (20). Entry in equilibrium is suboptimal because the monopolistically competitive firms fail to appropriate the entire surplus that flows from its entry.

And then after substituting in N from the above, the first order conditions for q and α are respectively

$$\lambda^{\frac{1-\lambda}{1-2\lambda}} \left(\frac{\left(1 - \lambda\right) H}{c_e} \frac{\overline{x}\left(q, \alpha\right)}{q} \right)^{\frac{1-\lambda}{2\lambda - 1}} \frac{\partial \left[\overline{x}\left(q, \alpha\right) / q \right]}{\partial q} - \frac{\partial \left[\left(\alpha - 1\right)^{\gamma} / q \right]}{\partial q} = 0, \tag{25}$$

[10] This verifies that the restriction in (A2) is necessary for the social optimum to be well-defined.

$$\lambda^{\frac{1-\lambda}{1-2\lambda}}\left(\frac{(1-\lambda)H}{c_e}\frac{\overline{x}(q,\alpha)}{q}\right)^{\frac{1-\lambda}{2\lambda-1}}\frac{\partial\left[\overline{x}(q,\alpha)/q\right]}{\partial\alpha}-\frac{\partial\left[(\alpha-1)^{\gamma}/q\right]}{\partial\alpha}=0. \qquad (26)$$

The difference between the above and the corresponding equilibrium conditions in (15) and (16) after the substitution from (19) and (20) is that $\lambda<1$ in the equilibrium conditions is replaced by $\lambda^{\frac{1-\lambda}{1-2\lambda}}>1$ in the optimality conditions. The distortions are entirely due to the non-marginal cost pricing and the suboptimal entry of firms in monopolistic competition that drive a wedge between the marginal revenue of firms and the shadow price of a unit of intermediate good

By combining (25) and (26), we see that the technology choice condition (21) is still satisfied in the social optimum. But now in lieu of the market tightness condition (22) , we have under social optimality:

$$\lambda^{\frac{1-\lambda}{1-2\lambda}}\left(\frac{H(1-\lambda)}{c_e}\left(1-\frac{1-e^{-\widehat{q}}}{\widehat{q}}\right)\right)^{\frac{1-\lambda}{2\lambda-1}}\left(2\frac{1-e^{-\widehat{q}}}{\widehat{q}}-1-e^{-\widehat{q}}\right)+ \left(\frac{1}{\widehat{q}}-\frac{1}{q}\right)^{\gamma-\frac{\lambda}{2\lambda-1}}q^{\gamma-1}=0 \qquad (27)$$

whose only difference with (22) is that its first term is augmented by $\lambda^{\frac{\lambda}{1-2\lambda}}>1$ instead of by $\lambda<1$.

Proposition 3 *In equilibrium, the job queue and capital intensity: q and α are sub-optimal, while $\widehat{q}$ rises above optimality.*

Proof. Replacing the factor λ in the equilibrium condition (22) by $\lambda^{\frac{1-\lambda}{1-2\lambda}}>\lambda$ has the same qualitative effects on q and $\widehat{q}$ as those from an increase in H. The results then follow from proposition 1. QED.

That the wedge between the marginal revenue and the shadow price instills firms with insufficient incentives to invest in physical capital is only to be expected. More to the point, when this happens, firms are also adopting a technology that is more forgiving in match imperfections and a shorter job queue suffices to allow them to find acceptable matches. Hence, more jobs are offered in equilibrium than in the social optimum. Even so, that $\widehat{q}$ is above its value in the social optimum means that the jobs are filled more often. Similarly, workers find jobs at a higher probability than they would in the social optimum as the jobs are less exact in its match requirement on top of there being more of them offered.

Although more jobs are created and each is filled at a higher probability in equilibrium than in optimality, aggregate output is too low. In particular,

Proposition 4 *Output per worker, expected output of intermediate good per vacancy and product variety are all suboptimal.*

Proof. In appendix A.

Again, the technology choice plays a crucial role in the conclusion that equilibrium employment is excessive. With or without the technology choice, monopolistic competition would drive a wedge between the firms' marginal revenue and the shadow price of a unit of intermediate good. Without the technology choice, the greater output may only be produced with more jobs created. With the technology choice and if assumption (A3) is satisfied, it is optimal for the greater output to be produced via a smaller number of the more capital intensive jobs to be created which are more exact in their skill requirement.

Perhaps the counter-argument that if the jobs created are more specialized, more of them should be created so that each worker has a greater probability of finding a good match sounds equally plausible. In the wage-posting model, this argument would not hold though as good matches may only be formed with a long job queue since it is the vacancies that attract workers. In a model where it is workers that attract job vacancies, it is conceivable that the counter-argument may indeed apply.

4. THE INFINITE HORIZON ECONOMY

I now generalize the analysis to an infinite horizon economy. Time is continuous and the interest rate is constant at some r. The matching of unemployed workers and vacant jobs take place at each moment of time. Unemployment in the steady state arises because a given worker–firm match is assumed to break down at some exogenous probability s. When the separation occurs, the worker returns to the unemployment pool and the capital installed for the job gets destroyed. All agents are risk neutral. Firms maximize expected discounted profit. Workers maximize expected discounted lifetime income. In this case, Acemoglu and Shimer (1999a) show that the discounted lifetime income of a currently unemployed worker is given by

$$u = \frac{\eta\left(q, \theta_c\right) w}{\left(r+s\right) q + \eta\left(q, \theta_c\right)} \tag{28}$$

in applying to job vacancies where his instantaneous probability of obtaining employment is $\mu = \eta\left(q, \theta_c\right)/q$. In equilibrium, unemployed workers must derive the same expected lifetime income from applying to any job vacancies. Then any triples of $\left(q, \theta_c, w\right)$ offered in equilibrium must satisfy

$$w = u^* \left(\left(r+s\right) \frac{q}{\eta\left(q, \theta_c\right)} + 1\right). \tag{29}$$

Let $J_i\left(t\right)$ be firm i's stock of unfilled vacancies and $E_i\left(t\right)$ its employment at time t. They satisfy

$$\dot{E}_i\left(t\right) = \eta\left(q, \theta_c\right) J_i\left(t\right) - sE_i\left(t\right). \tag{30}$$

The departure of $sE_i\left(t\right)$ workers at each t lowers the firm's productive capacity by $sE_i\left(t\right)\widehat{x}_i\left(t\right) = sy_i\left(t\right)$, while the recruitment of $\eta\left(q, \theta_c\right) J_i\left(t\right)$ workers would augment it by $\overline{X}\left(q, \theta_c, \alpha\right) J_i\left(t\right)$. This implies

$$\dot{y}_i\left(t\right) = \overline{X}\left(q, \theta_c, \alpha\right) J_i\left(t\right) - sy_i\left(t\right). \tag{31}$$

Denote the number of new vacancies created by the firm at time t by $j_i(t)$. The stock of unfilled vacancies evolves according to

$$\dot{J}_i(t) = j_i(t) - \eta(q, \theta_c) J_i(t). \tag{32}$$

At each t, the firm's cash flow is

$$\begin{aligned} \pi_i(t) &= y(t)^{1-\lambda} y_i(t)^{\lambda} - wE_i(t) - k(t)^{\gamma} j_i(t) \\ &= y(t)^{1-\lambda} y_i(t)^{\lambda} - wE_i(t) - (\alpha - 1)^{\gamma} j_i(t). \end{aligned} \tag{33}$$

Taking aggregate output $y(t)$ as given, the firm maximizes discounted profit

$$V_i = \int_0^{\infty} e^{-rt} \pi_i(t)\, dt$$

subject to $(29) - (32)$.

Lemma 5 *In the steady state, the solution of the above optimal control problem is characterized by*

$$y_i = y \left(\frac{\lambda \overline{X}}{w\eta + (\alpha - 1)^{\gamma} (\eta + r)(r + s)} \right)^{\frac{1}{1-\lambda}}. \tag{34}$$

$$(r + s)\eta\gamma(\alpha - 1)^{\gamma - 1} - \frac{\partial \overline{X}}{\partial \alpha} \frac{1}{\overline{\overline{X}}} \left(w\eta + (\alpha - 1)^{\gamma}(\eta + r)(r + s) \right) = 0, \tag{35}$$

$$\frac{\partial \eta}{\partial \theta_c} \left((\alpha - 1)^{\gamma} - u^* \frac{r}{s} \frac{q}{\eta} + \frac{u^*}{r + s} \right) - \frac{\partial \overline{X}}{\partial \theta_c} \frac{1}{\overline{\overline{X}}} \left(\frac{w\eta}{r + s} + (\alpha - 1)^{\gamma}(\eta + r) \right) = 0, \tag{36}$$

$$\begin{aligned} &\frac{\partial \eta}{\partial q} \left((\alpha - 1)^{\gamma} - u^* \frac{r}{s} \frac{q}{\eta} + \frac{u^*}{r + s} + u^* \frac{r + s}{s} \left(\frac{\partial \eta}{\partial q} \right)^{-1} \right) - \\ &\qquad \frac{\partial \overline{X}}{\partial q} \frac{1}{\overline{\overline{X}}} \left(\frac{w\eta}{r + s} + (\alpha - 1)^{\gamma}(\eta + r) \right) = 0. \end{aligned} \tag{37}$$

Proof. In appendix A.

We have from (29) and (35) − (37) four equations in w, q, α and θ_c as functions of u^* which is pinned down by the free entry condition that each firm may only earn zero discounted profit in equilibrium. To derive this condition, first notice that from (30) − (32), we have in the steady state

$$\eta J_i = sE_i, \tag{38}$$

$$j_i = \eta J_i, \tag{39}$$

$$\overline{X} J_i = s y_i. \tag{40}$$

Hence the firm's expense at each t may be expressed as

$$wE_i(t) - (\alpha - 1)^{\gamma} j_i(t) = \frac{y_i}{\overline{X}} \left(w\eta + (\alpha - 1)^{\gamma} s\eta \right)$$

And using (34), the cash flow from (33) is given by

$$y \left(\frac{\lambda \overline{X}}{w\eta + (\alpha - 1)^{\gamma} (\eta + r)(r + s)} \right)^{\frac{\lambda}{1-\lambda}} \left(1 - \lambda \frac{w\eta + (\alpha - 1)^{\gamma} \eta s}{w\eta + (\alpha - 1)^{\gamma} (\eta + r)(r + s)} \right) = rc_e \tag{41}$$

which is equal to the interest expense of entry in equilibrium.

To complete the solution of the equilibrium, I now turn to the determination of aggregate output y. In the steady state, job creation is equal to job destruction at each point in time. Let u be the rate of unemployment

$$JC = uH\frac{\eta}{q} = s(1 - u)H = JD.$$

Solving for u

$$u = \frac{sq}{\eta + sq}, \tag{42}$$

and this implies an aggregate employment of

$$\widetilde{E} = H\frac{\eta}{\eta + sq}.$$

With output of intermediate good per employed worker still given by $\overline{X}/\eta$, aggregate output is equal to

$$y = N^{\frac{1}{\lambda}} \frac{H}{N} \frac{\overline{X}}{\eta + sq}. \tag{43}$$

By definition, the job queue is

$$q = \frac{uH}{NJ_i}.$$

Using (42) for u and (34) and (40) for J_i, the above becomes

$$y = \frac{H}{N} \frac{\overline{X}}{\eta + sq} \left(\frac{w\eta + (\alpha - 1)^{\gamma} (r + s) (\eta + r)}{\lambda \overline{X}} \right)^{\frac{1}{1-\lambda}}. \tag{44}$$

Equations (43) and (44) imply

$$N = \left(\frac{w\eta + (\alpha - 1)^{\gamma} (r + s) (\eta + r)}{\lambda \overline{X}} \right)^{\frac{\lambda}{1-\lambda}}$$

from which it follows from (43) that

$$y = H \frac{w\eta + (\alpha - 1)^{\gamma} (r + s) (\eta + r)}{\lambda (\eta + sq)}. \tag{45}$$

We now have the complete characterization of the equilibrium given by (29), (35) – (41) and (45).

The system is too complicated to be subject to a qualitative analysis. And we may only study the properties of the equilibrium through numerical experiments. As benchmark parameter values for the following, I set $r = s = 0.05$, $c_e = 1$, $\gamma = 1.5$ and $\lambda = 0.8$ where the values for γ and λ are chosen to satisfy assumption (A3). The comparative steady states are summarized in the various panels of fig.3.

First, capital intensity is increasing in market size, just as in the two–period economy. However, the second panel shows that the job queue is now falling with market size. Evidently, assumption (A3) is not sufficient to guarantee that the positive effect on the job queue of more capital intensive jobs being created dominates the negative effect on the job queue of job creation becoming more profitable. However, this does

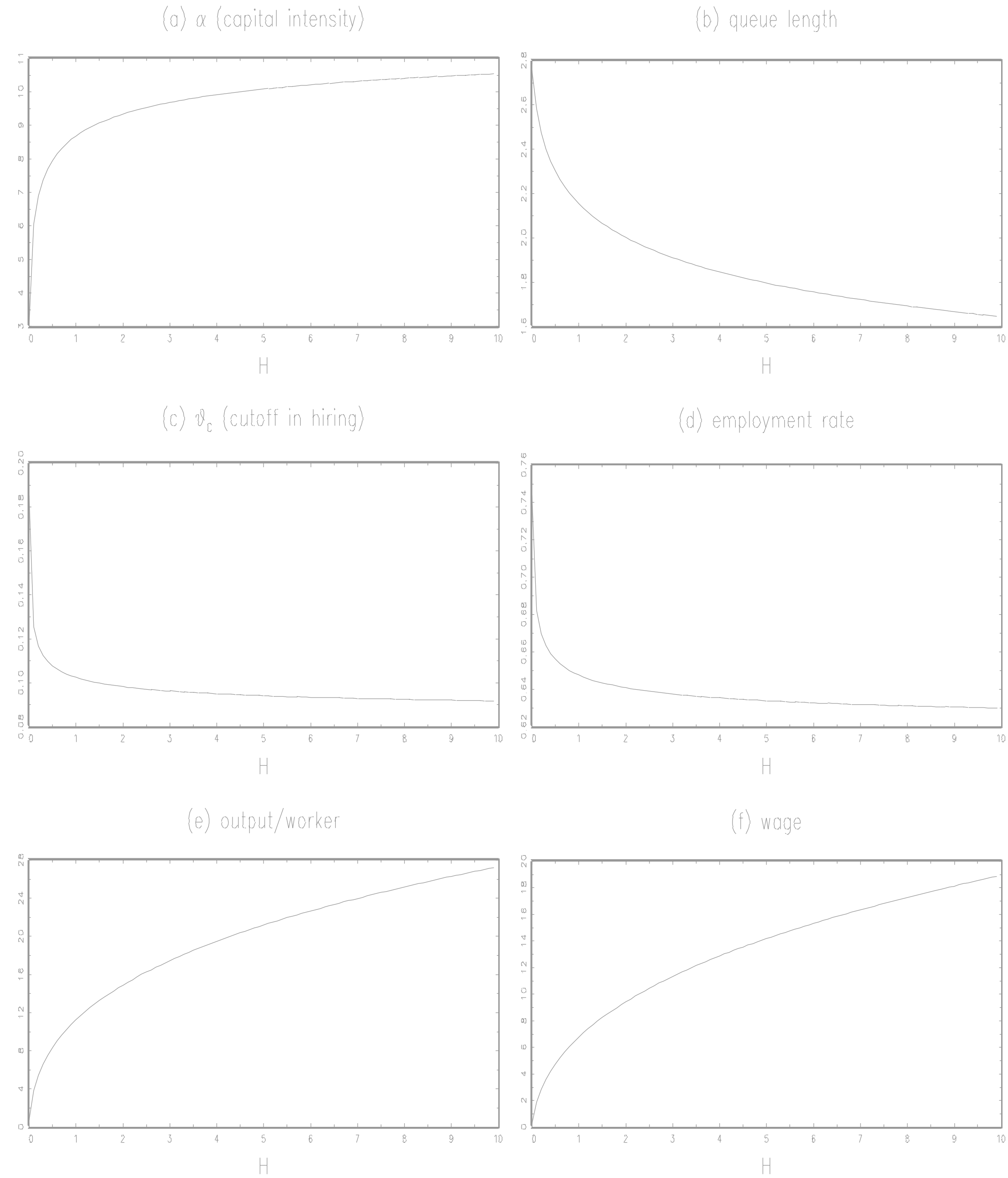

FIG. 3. Comparative steady states

not imply that there would be greater employment in the steady state. With more capital intensive and specialized jobs created, firms are obliged to be more selective in accepting job applications as evidenced by the downward sloping relationship between θ_c and market size in panel (c). In the present case, the steady state employment indeed falls with market size as the effect of more exact match requirement on employment dominates the opposing effect of the availability of more job vacancies (shorter queue length). However, the falling employment is accompanied by greater output per worker and a higher wage rate as the last two panels testify. Not shown is that the utility of an unemployed worker as well as the product variety are also increasing in market size. In all, the effects of market size on labor market outcomes and productivity are essentially identical to the effects in two-period economy, with the only difference being how the length of the job queue varies with market size.

In the infinite horizon economy, the steady state equilibrium depends, among other exogenous variables, on the interest rate and the rate of job destruction–two considerations that do not apply previously in the two–period economy. It is thus not surprising that the restrictions on γ and λ imposed by assumption (A3) do not guarantee that the job queue would lengthen with market size even though they do in the two–period economy. It is only when r and s are also chosen appropriately that we may establish the same conclusion. Intuitively, the result would obtain when the productivity gains from greater product variety spur a major increase in the capital intensity of the jobs offered. For then the effect of the rising match requirement would dominate the market tightness effect. A smaller interest rate turns the investment decision more elastic to the increase in the marginal revenue under which the capital intensity effect should be stronger and be more likely to dominate.

The effect of s is harder to evaluate as it should affect investment incentives and market tightness in the same way. On the one hand, a larger s should make the investment decision less elastic to changes in the marginal revenue for the payoff to create capital intensive jobs should diminish when these jobs are less likely to last. But the same force should apply equally to the market tightness effect since any jobs created, capital intensive or otherwise, are less likely to last in any case.

My numerical experiments suggest that it is at a larger s that the capital intensity effect dominates. Holding s at the benchmark of 0.05, I find that the job queue is still decreasing in market size for an interest rate as small as $r = 0.01$. However, when s is allowed to increase to 0.15, the relationship between the job queue and market size is turned around for an interest rate that may be as high as $r = 0.03$. The comparative steady states are reported in the various panels of fig. 4 for this combination of $\{r, s\} = \{0.03, 0.15\}$. The qualitative results are essentially identical to the benchmark, except that in panel (b), the queue length is now increasing in market size, just as in the two–period economy when assumption (A3) is satisfied. More interestingly, when compared to the results in fig.3, we can see that at each H, the job queue is longer. In other words, fewer jobs (per unemployed) are offered. The lowering of the interest rate quite unexpectedly does not help raise employment. The beneficial effect is solely reflected in the increase in the capital intensity of the jobs created. But wages are higher as a result as evidenced by a comparison between the last panels in fig.3 and fig.4. In fact, we can see from comparing panels (e) of fig.3 and fig.4 that the increase in the capital intensity of the jobs created suffices to raise output per worker even though employment has fallen.

The effects of the interest rate on how employment may respond to market size as well as on employment itself should work in the opposite direction too. At a higher interest rate, the investment decision becomes less elastic to changes in the marginal revenue. Not only that the job queue may shorten as market size increases just as in the benchmark, but also employment may rise if the market tightness effect dominates the now weakened capital intensity effect. To verify the intuition, I repeat the comparative steady states with an interest rate $r = 0.2$. The results are reported

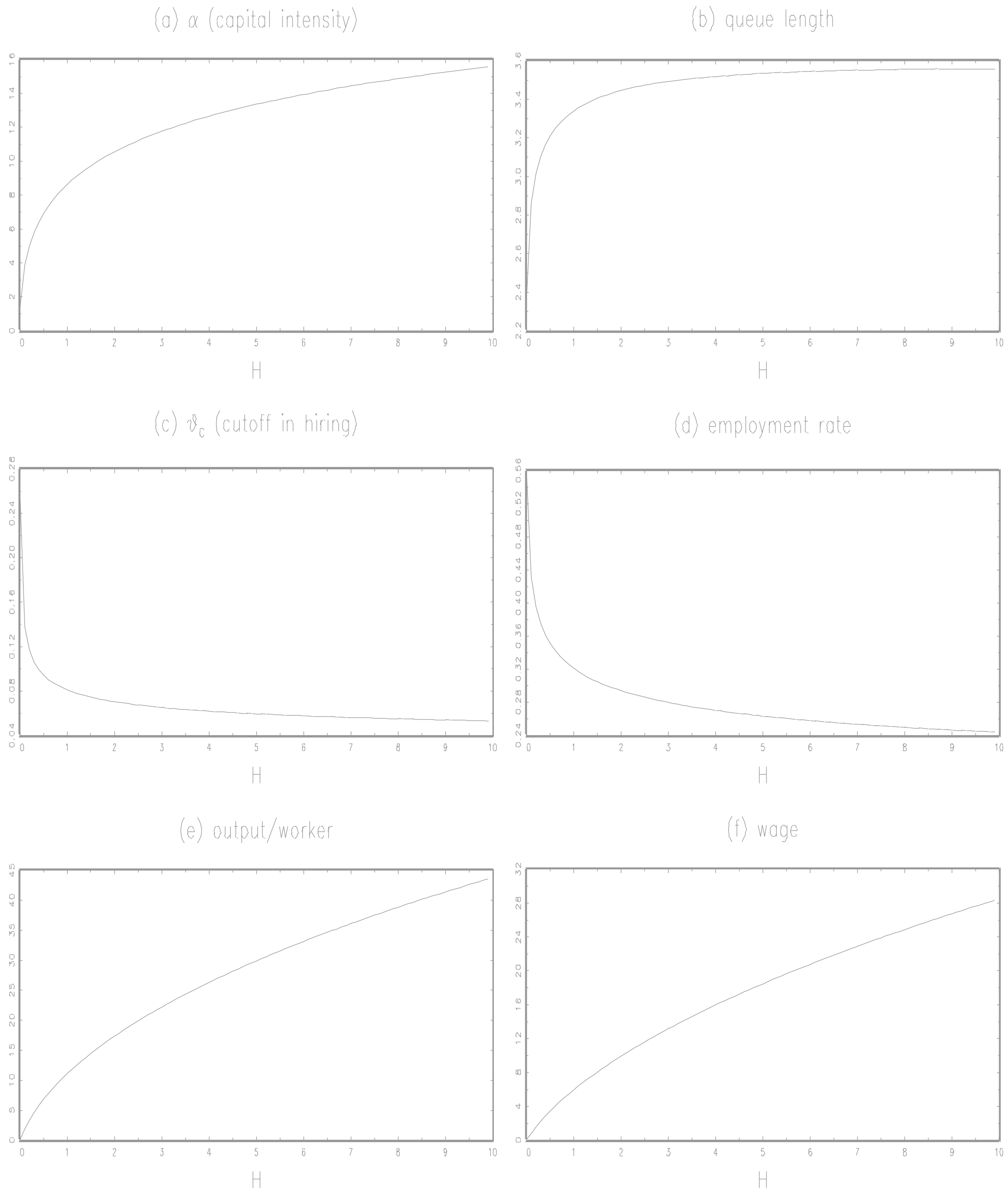

Fig. 4. Comparative steady states with increasing job queue

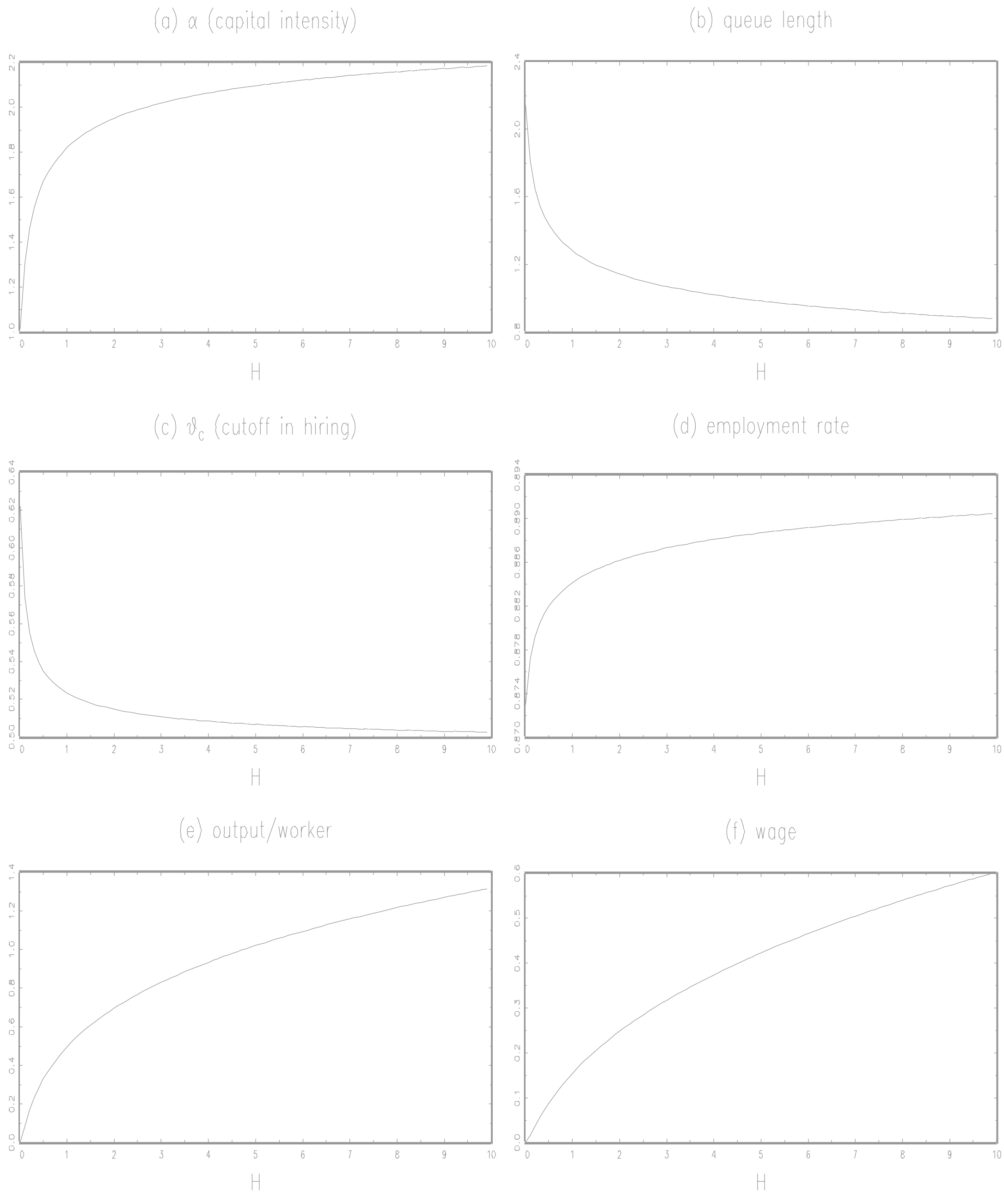

Fig. 5. Comparative steady states: increasing employment

in the various panels of fig.5. At this high interest rate, the job queue is falling with market size, a return to the relationship in the benchmark. Furthermore, the greater job creation now in fact results in greater employment. The rising employment however does not make workers better off. At each H, wages and output per worker are lower when compared to the benchmark shown in fig.3. The effects of the interest rate on employment and output are similar to the effects of market size. Once again, output and employment may deviate in opposite directions as a low interest rate may raise output but not necessarily employment.

5. CONCLUDING REMARK

The policy implication that monopolistic competition, accompanied by the technology choice on the specificity of jobs, results in excessive equilibrium employment deserves further scrutiny. The conclusion is derived under the parameter restrictions in assumption A3, which also guarantee that increases in market size will lower employment. The two results are closely related. In the latter, the productivity gains from greater product variety raise marginal revenue and induce firms to create more specialized jobs. In similar manner, the planner would like to see more specialized jobs to be created to raise aggregate output above equilibrium in order to close the wedge between the shadow price of a unit of intermediate good output and the marginal revenue to the monopolistically competitive firms. Both the firms and the planner choose to raise output through increasing the capital intensity and the skill specificity of the jobs to the extent that a longer job queue is necessary to sustain their viability. This is due to assumption A3, which however is only a sufficient but not a necessary condition for the two results to hold. Ideally, one would like to derive the necessary and sufficient condition for each result and compare them to see if they are identical as is appeared to be the case intuitively. But conditions that apply in the two–period economy do not necessarily apply in the infinite horizon economy as evidenced by the fact that A3 guarantees neither a positive relationship between the job queue and market size nor between the employment rate and market size, two results that are certain to hold previously in the two–period economy. Hence, the task should really be directed toward the derivations of these conditions in the infinite horizon economy. The complications involved would lengthen this paper considerably and I may only leave them for future research.

APPENDIX

A. Proofs

Lemma 1.—

Substitute (3) into (2) and then into (4)

$$\begin{aligned} X_n &= \sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} \times \\ &\quad k\left\{1-(1-\theta_c)^n - \alpha\left((1+n)^{-1} - (1+n)^{-1}(1-\theta_c)^{1+n} - (1-\theta_c)^n \theta_c\right)\right\}. \end{aligned} \tag{46}$$

Now evaluate the summations of each term inside the curly bracket separately:

$$\sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} \left(1-(1-\theta_c)^n\right) = 1-(1-p\theta_c)^h,$$

$$\sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} \frac{1}{1+n} = \frac{1-(1-p)^{h+1}}{p(h+1)} - (1-p)^h,$$

$$\sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} \frac{(1-\theta_c)^{1+n}}{1+n} = \frac{(1-p\theta_c)^{h+1} - (1-p)^{h+1}}{p(h+1)} - (1-\theta_c)(1-p)^h,$$

$$\sum_{n=1}^{h} \binom{h}{n} p^n (1-p)^{h-n} (1-\theta_c)^n \theta_c = \theta_c\left((1-p\theta_c)^h - (1-p)^h\right).$$

Combining the above into (46), let $p = \frac{1}{J}$ and $q = \frac{h}{J}$ and take limit as $h \to \infty$ yields part (a). Part (b) is simply the limit of the first line above when $h \to \infty$.

Lemma 2.—

Proof. Hosios (1990) shows that a worker that applies to a vacancy that attracts an average queue length of q would get hired with probability $\frac{1-e^{-q}}{q}$ if the firm randomly picks one applicant among the possibly many applicants that it has attracted. Lemma 1b shows that a vacancy that attracts an average queue length of q with a minimum match requirement of $\theta \leq \theta_c$ is as if it has an average queue length of $q\theta_c$ that hires any $\theta \leq 1$. Suppose the match between the given worker and the firm satisfies $\theta \leq \theta_c$. The worker then has an equal probability among the other qualified applicants to be the best match and ex-ante, it is as if the firm would pick the worker with equal probability as others. Therefore conditional on the worker qualifies as an acceptable match, he would obtain employment with probability $\frac{1-e^{-q\theta_c}}{q\theta_c}$. Since the worker qualifies as an acceptable match with probability θ_c, $\mu(q, \theta_c) = \eta(q, \theta_c)/q$.

Lemma 3.—

Part (i) is from simple differentiation. Parts (ii) and (iii) are from applying the L'Hospita rule.

Lemma 4.—

Under assumption (A3), the left side of (22) is decreasing in $\widehat{q}$ and increasing in q. If the equation defines an implicit function for $q = F(\widehat{q})$, $F(\widehat{q})$ is an increasing function. Clearly the solution of q to (22) must exceed $\widehat{q}$. Then $F(\widehat{q})$ if it exists is positive valued. The first term of (22) is negative and independent of q, while the second term is increasing in q without bound and tends to 0 as $q \to \widehat{q}$. This implies a unique solution of $q > \widehat{q}$ exists. As $\widehat{q} \to 0$, the first term of (22) vanishes and the second term may only vanish too if $q \to 0$. Finally, as $\widehat{q} \to \infty$, $\frac{1-e^{-\widehat{q}}}{\widehat{q}} \to 0$ so that the first term of (22) tends to a constant while the second term increases without bound for fixed q and so the solution for q too must increase without bound.

Proposition 2.—

The market tightness condition may be rewritten as

$$\lambda\left(\frac{H(1-\lambda)}{c_e}\frac{\overline{x}(q,\alpha)}{q}\right)^{\frac{1-\lambda}{2\lambda-1}}\left(2\frac{\alpha}{q}\left(1-e^{-q/\alpha}\right)-1-e^{-q/\alpha}\right)+(\alpha-1)^{\gamma-1}=0. \quad (47)$$

From proposition 1, as H increases, $\widehat{q}=q/\alpha$ falls and α increases. Then the term $\lambda\left(\frac{H(1-\lambda)}{c_e}\frac{\overline{x}(q,\alpha)}{q}\right)^{\frac{1-\lambda}{2\lambda-1}}$ in the equation above must have increased since

$$\left(2\frac{\alpha}{q}\left(1-e^{-q/\alpha}\right)-1-e^{-q/\alpha}\right)<0$$

falls and $(\alpha-1)^{\gamma-1}$ in the same equation rises as they are decreasing in $\widehat{q}$ and increasing in α respectively. By virtue of (19) and (20), this implies that y and N are increasing in H. It follows from the maximization in (14) that this raises u^* and $\overline{x}/q$. The latter increases would raise $\overline{x}$ as q has risen in the interim. Output per worker from

$$\frac{y}{H}=N^{\frac{1}{\lambda}-1}\frac{\overline{x}}{q}$$

must increase too as N and $\overline{x}/q$ have both risen. The increases in w follows trivially from the increase in expected utility being accompanied by the decline in the probability of employment.

Proposition 4.—

Equations (19) and (14) imply that for given N, the equilibrium maximizes

$$u^*=\max_{\{q,\alpha\}}\left\{\lambda N^{\frac{1-\lambda}{\lambda}}\frac{\overline{x}(q,\alpha)}{q}-\frac{(\alpha-1)^{\gamma}}{q}\right\} \quad (48)$$

where from (19) and (20).

$$\lambda N^{\frac{1-\lambda}{\lambda}}=\lambda\left(\frac{H(1-\lambda)}{c_e}\frac{\overline{x}(q,\alpha)}{q}\right)^{\frac{1-\lambda}{2\lambda-1}}. \quad (49)$$

Equations (24) and (23) imply that the social optimum maximizes

$$s=\max_{\{q,\alpha\}}\left\{N^{\frac{1-\lambda}{\lambda}}\frac{\overline{x}(q,\alpha)}{q}-\frac{(\alpha-1)^{\gamma}}{q}\right\} \quad (50)$$

where from (24)

$$N^{\frac{1-\lambda}{\lambda}} = \lambda^{\frac{1-\lambda}{1-2\lambda}} \left(\frac{(1-\lambda) H}{c_e} \frac{\overline{x}(q,\alpha)}{q} \right)^{\frac{1-\lambda}{2\lambda-1}}. \tag{51}$$

Denote values for the social optimum with the superscript o and equilibrium with $*$. If $\widehat{q}^o < \widehat{q}^*$ and $q^o > q^*$ as established by proposition 3, then (47) and (25) imply

$$\lambda^{\frac{1-\lambda}{1-2\lambda}} \left(\frac{(1-\lambda) H}{c_e} \left(\frac{\overline{x}}{q}\right)^o \right)^{\frac{1-\lambda}{2\lambda-1}} > \lambda \left(\frac{H(1-\lambda)}{c_e} \left(\frac{\overline{x}}{q}\right)^* \right)^{\frac{1-\lambda}{2\lambda-1}}$$

This implies from (48) – (50) that $\left(\frac{\overline{x}}{q}\right)^o > \left(\frac{\overline{x}}{q}\right)^*$ and therefore $\overline{x}^o > \overline{x}^*$. Furthermore, from (20) and (24), $N^o > N^*$. Hence output per worker must have been lower in equilibrium than in the social optimum.

Lemmas 5 and 6.—

The Hamiltonian of the control problem is

$$\begin{aligned} \mathcal{H}(t) &= e^{-rt} \left(y(t)^{1-\lambda} y_i(t)^{\lambda} - wE_i(t) - (\alpha-1)^{\gamma} j_i(t) \right) + \\ & \psi_1(t) \left(\eta(q,\theta_c) J_i(t) - sE_i(t) \right) + \psi_2(t) \left(\overline{X}(q,\theta_c,\alpha) J_i(t) - sy_i(t) \right) + \\ & \psi_3(t) \left(j_i(t) - \eta(q,\theta_c) J_i(t) \right). \end{aligned}$$

The evolution of the co-state variables ψ_1, ψ_2 and ψ_3 satisfy respectively:

$$-e^{-rt} w - \psi_1 s = -\dot{\psi}_1, \tag{52}$$

$$e^{-rt} \lambda y^{1-\lambda} y_i^{\lambda-1} - \psi_2 s = -\dot{\psi}_2, \tag{53}$$

$$\psi_1 \eta + \psi_2 \overline{X} - \psi_3 \eta = -\dot{\psi}_3 . \tag{54}$$

The first order condition with respect to j_i is

$$-e^{-rt} (\alpha-1)^{\gamma} + \psi_3 = 0, \tag{55}$$

In the steady state, α (and so as q and θ_c) stays constant. Take time derivative of the above

$$re^{-rt} (\alpha-1)^{\gamma} + \dot{\psi}_3 = 0. \tag{56}$$

Then use $(52)-(55)$ to eliminate the co-state variables:

$$\lambda y^{1-\lambda} y_i^{\lambda-1} \overline{X} - w\eta - (\alpha-1)^{\gamma} (\eta + r)(r+s) = 0 \tag{57}$$

which implies (34) in the text.

Next we derive the first order condition of the Hamiltonian with respect to α

$$-e^{-rt}\gamma (\alpha-1)^{\gamma-1} j_i + \psi_2 J_i \frac{\partial \overline{X}}{\partial \alpha} = 0. \tag{58}$$

Assuming the steady state (so that all state and control variables stay constant over time), take time derivative of the above and then use $(52)-(56)$ and (39) to eliminate the co-state variables and j_i.

$$(r+s)\,\eta\gamma (\alpha-1)^{\gamma-1} - \lambda y^{1-\lambda} y_i^{\lambda-1} \frac{\partial \overline{X}}{\partial \alpha} = 0.$$

Finally use (57) to substitute out $\lambda y^{1-\lambda} y_i^{\lambda-1}$ yields (35).

The first order conditions of the Hamiltonian with respect to θ_c and q are respectively

$$-e^{-rt} E_i \frac{\partial w}{\partial \theta_c} + \psi_1 J_i \frac{\partial \eta}{\partial \theta_c} + \psi_2 J_i \frac{\partial \overline{X}}{\partial \theta_c} - \psi_3 J_i \frac{\partial \eta}{\partial \theta_c} = 0,$$
$$-e^{-rt} E_i \frac{\partial w}{\partial q} + \psi_1 J_i \frac{\partial \eta}{\partial q} + \psi_2 J_i \frac{\partial \overline{X}}{\partial q} - \psi_3 J_i \frac{\partial \eta}{\partial q} = 0.$$

Following the same procedure to derive (35) from (58) and then using (29) to evaluate $\partial w/\partial \theta_c$ and $\partial w/\partial q$ turn the above into (36) and (37).

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